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Long Memory and Structural Changes in the Forward Discount: An Empirical Investigation

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Abstract

Many empirical studies find a negative correlation between the returns on the nominal spot exchange rate and the lagged forward discount. This forward discount anomaly implies that the current forward rate is a biased predictor of the future spot rate. A large number of studies in the existing literature try to explain this anomaly, and recent work has tried to explain the anomaly as a statistical artifact based on (1) the long memory behavior of the forward discount; or (2) the existence of structural breaks in the forward discount. In this paper, we evaluate the evidence for long memory and structural change in the forward discount. Our approach is as follows. First, we nonparametrically estimate the long memory parameter for a number of forward discount series without allowing for structural breaks. Second, we test for and estimate a multiple mean break model and then adjust for the structural breaks in the forward discount. Finally, we re-estimate the long memory parameter on the mean-break adjusted data. We show that allowing for structural breaks drastically reduces the persistence of the forward discount. However, after removing the breaks, we still find evidence of stationary long memory in all of the forward discount series. Our results have important implications for understanding the statistical properties of the forward discount, because we confirm not only the presence of long memory behavior in the forward discount but also the importance of structural breaks.

Key Words : Long Memory, Structural Changes, Forward Discount

JEL code : C14, C22, F31

1. Introduction

Many empirical studies find a negative correlation between the returns on the nominal spot exchange rate and the lagged forward discount. This forward discount anomaly implies that the current forward rate is a biased predictor of the future spot rate. A large number of studies in the existing literature try to explain this anomaly. Engel (1996) summarized four explanations: (1) existence of a foreign exchange risk premium; (2) a peso problem, (3) irrational expectations; (4) international financial market inefficiency from various frictions. In two detailed studies, Baillie and Bollerslev (1994, 2000) focused on the time series properties of the spot rate and forward discount as an explanation for the forward discount anomaly. They argued that the forward discount anomaly is due to the statistical properties of the data, because the forward discount is a fractionally integrated (long memory) process and the rate of return on the spot exchange rate is a stationary process which creates an unbalanced test regression. Maynard and Phillips (2001) provided similar results as Baillie and Bollerslev. They argued that traditional asymptotic theory may not be applicable to test forward rate unbiasedness due to the fractional integration of the forward discount and they propose a new limit theory. Their limit theory for the FRUH test statistics has nonstandard limiting distributions with long left tails, which may explain the forward discount anomaly as a statistical artifact.

A criticism against models of long memory is that the long memory property in the data may be due to the presence of structural breaks or regime switches. This is called "the spurious long memory process." Several recent works including Granger (1999), Granger and Hyung (1999), and Diebold and Inoue (2001), show that structural breaks or regime switching can generate spurious long memory behavior in an observed time series. Indeed, Sakoulis and Zivot (2001) find evidence for structural breaks in the mean and variance of the forward discount, and argue that these breaks could be caused by events like discrete changes in policy and changes in interest rates due to the business cycle. After correcting for multiple structural breaks in the mean of the forward discount, they find the persistence of the forward discount is substantially reduced. The focus of this paper is to expand on the analysis of Sakoulis and Zivot and critically evaluate the evidence for long memory and structural breaks in the forward discount.

In practice, the usual method to estimate the long memory parameter *'d'* characterizing a time series is the nonparametric log periodogram regression estimator suggested by Geweke and Porter-Hudak (1983). When we estimate the long memory parameter using the log periodogram regression, we first difference the data. This estimator is appropriate for stationary long memory process with $-0.5 < d < 0.5$. However, Agiakloglou *et al.* (1993) show that the estimator is not invariant to first differencing, so that there might be bias due to over-differencing of the data. Kim and Phillips (1999, 2000) suggest that if we have no prior information about the magnitude of the long memory parameter before estimation, we need a more flexible estimation technique and inference for both stationary and nonstationary cases. They propose to estimate d using a modified log periodogram (hereafter MLP) regression estimator that includes the nonstationary range where $d \geq 0.5$.

There is a large literature on structural break models, but there are only a few recent studies that deal with multiple structural breaks, and even fewer dealing with long memory and multiple structural breaks[1]. In this paper, we assume that the potential structural break dates are unknown and we follow Bai and Perron (1998, 2003) and estimate the unknown break dates using the least squares principle. We consider a structural change in mean model that allows the errors to be serially correlated and heteroskedastic.

Our approach is as follows. First, we estimate the long memory parameter for a number of forward discount series using the MLP regression without allowing for structural breaks. Second,

[1] Sibbertsen (2001) surveys some of the issues associated with distinguishing long memory processes from some simple structural break models.

we test for and estimate the multiple mean break model using Bai and Perron's method, and then adjust for the structural breaks in the forward discount. Finally, we re-estimate the long memory parameter using the MLP regression on the mean-break adjusted data.

We show that allowing for structural breaks drastically reduces the persistence of the forward discount. However, after removing the breaks, we still find evidence of stationary long memory behavior in all of the forward discount series. Our results have important implications for understanding the statistical properties of the forward discount, because we confirm not only the presence of long memory in the forward discount but also the importance of structural breaks.

The remainder of this paper is organized as follows. Section 2 provides a brief review of the literature relating the forward discount anomaly, long memory and structural breaks. Section 3 reviews some properties of long memory processes and defines the MLP regression estimator of Kim and Phillips. Section 4 presents the multiple mean break model and reviews Bai and Perron's methodology to test for and estimate multiple structural breaks. Section 5 gives the empirical results, and Section 6 concludes with the implications of our findings.

2. The Forward Discount Anomaly: Long Memory and Structural Breaks

The uncovered interest rate parity and covered interest parity imply that the current forward rate is an unbiased predictor of the future spot rate. Covered interest parity implies

$$\frac{F_t}{S_t} = \frac{1+i_t}{1+i_t^*}, \tag{1}$$

where i_t denotes the monthly interest rate on one month home country risk free bond, i_t^* denotes the monthly interest rate on one month foreign country risk free bond, S_t denotes the spot exchange rate and F_t denotes the monthly forward exchange rate, the price in foreign currency units of the dollar deliverable 30 days from *t*. We can rewrite equation (1) as

$$\frac{F_t - S_t}{S_t} = \frac{i_t - i_t^*}{1+i_t^*}. \tag{2}$$

In logs, the relationship is approximately

$$f_t - s_t = i_t - i_t^*, \tag{3}$$

where $f_t = \log(F_t)$ and $s_t = \log(S_t)$.

Assuming rational expectations and risk neutrality, we obtain the uncovered interest parity equation

$$\frac{E_t(S_{t+1}) - S_t}{S_t} = \frac{i_t - i_t^*}{1+i_t^*}, \tag{4}$$

which implies foreign exchange market efficiency. In logs, this relationship is approximately

$$E_t(s_{t+1}) - s_t = i_t - i_t^*, \tag{5}$$

where $E_t(\cdot)$ denotes expectation conditional on information available at time *t*. From equations (3) and (5), we find that the forward rate is an unbiased predictor of the future spot exchange rate, $E_t(s_{t+1}) = f_t$. We call this as the forward rate unbiasedness hypothesis (FRUH).

In practice, we test the FRUH using the regression[2]

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1}, \tag{6}$$

where $f_t - s_t$ is the forward discount. The null hypothesis that the FRUH holds is α =0, β =1 and $E_t(\varepsilon_{t+1})$ =0 which implies that $E_t(s_{t+1}) = f_t$. However, the typical empirical finding is that FRUH is not only rejected but also that estimation of equation (6) provides a significantly negative β. This anomalous empirical finding is often referred to as the "forward discount anomaly".

Several approaches have been taken to explain the forward discount anomaly. As noted earlier, Engel (1996) suggests possible explanations and focuses on a time varying rational expectations risk premium. In this paper, we focus on the time series properties of the forward discount in the same sense as Baillie and Bollerslev (1994, 2000), Maynard and Phillips (2001) and Sakoulis and Zivot (2001).

Numerous studies show that the spot and forward exchange rates are both *I*(1) processes so that the difference of the spot and forward exchange rates are stationary *I*(0) variables. Therefore, the time series behavior of spot and forward exchange rates implies restrictions on the behavior of the forward discount and the rational expectations risk premium, it is exists.

To see this, consider the following decomposition for the forward exchange rate due to Fama (1984):

$$f_t = s_{t+1} + rp_{t+1} + u_{t+1}, \tag{7}$$

[2] See Engel (1996) and Zivot (2000) for a review.

where rp_{t+1}, u_{t+1} are next period's rational expectations risk premium and rational expectations forecast error term. We can simply rewrite $s_{t+1} = s_t + \Delta s_{t+1}$ and plug into (7) to get

$$f_t - s_t = \Delta s_{t+1} + rp_{t+1} + u_{t+1}. \tag{8}$$

In (8), the change in the spot exchange rate and the forecast error are stationary. Now, suppose the forward discount is $I(1)$. Then rp_{t+1} will be an $I(1)$ process as well. Evans and Lewis (1995), however, argue that a unit root risk premia would be very hard to rationalize since most economic models of the risk premium imply it depends on other stationary time series. Nonetheless, Crowder (1994) tested for a unit root in a number of monthly forward discount series from 1974 to 1991 and failed to reject a unit root using augmented Dickey-Fuller tests. However, it is well known that these tests have very low power against the alternative of fractional integration. Additionally, he rejected the null that the forward discount is $I(0)$ using the KPSS test of Kwiatowski, Phillips, Schmidt and Shin (1992).

Baillie and Bollerslev (1994) argued that Crowder's results do not necessarily guarantee the existence of a unit root in the forward discount. They used the same data as Crowder (1994) and compared the autocorrelations among the spot exchange rate, the return on the spot exchange rate, and the forward discount. Because there is strong evidence that the spot exchange has a unit root, comparing the correlograms provides good intuition about the properties of the forward discount. They find the degree of persistence of the forward discount's autocorrelations is drastically less than the counterparts of the spot rate. They also estimated ARFIMA models and reported point estimates for d, the order of fractional integration, equal to 0.77, 0.45, and 0.55 for Germany, Canada, and the U.K., respectively.[3] The last two cases have the properties of mean reversion with infinite variance. These results suggest that we can model the forward discount process as a

[3] Cheung (1993) find the evidence of long memory in the log of exchange rate changes for the monthly data from 1974 to 1989 of German Mark, Swiss Franc, French Franc, and Japanese Yen except very marginal result of British Pound.

fractionally integrated process and the spot rates as $I(1)$; therefore, the risk premium should be fractionally integrated.[4]

More recently, Baillie and Bollerslev (2000) suggested that the forward anomaly is not as bad as we think. This is at least partly due to statistical properties of the data, such as the very persistent autocorrelation in the forward discount. They showed, using Monte Carlo simulations, that $\hat{\beta}$ in the differences regression (6) will converge very slowly to its true value of unity. They also argued that the slow decay of the autocorrelations of the forward discount exacerbates the finite sample bias.[5]

Maynard and Phillips (2001) also argued that the forward discount is dominated by a nonstationary long memory component. Using Kim and Phillips' MLP regression, their estimates of d based on daily data are quite a bit larger than those of Baillie and Bollerslev (1994), who use monthly data. All estimates are in the range $0.882 \leq d < 1$, which suggest that the forward discounts have non-stationary properties. They go on to explain the forward discount anomaly as a statistical anomaly due to the nonstandard limit distributions resulting from estimating a regression equation with a stationary dependent variable and a nonstationary fractionally integrated regressor.

The results of Crowder, Baillie and Bollerslev, and Maynard and Phillips, while compelling, lack a degree of economic motivation. The usual explanation for long memory behavior in economic time series is based on the result that aggregation of independent weakly dependent series can produce a strongly dependent series, see Granger (1980) and Lobato and Savin (1998). This aggregation argument is not very compelling for the forward discount since it may be interpreted as an interest rate differential.

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[4] This implies that spot and forward rates are fractionally cointegrated. However, when the forward discount is a fractionally integrated process, Engel (1996) points out that the risk neutral efficient market hypothesis must be rejected since the forward discount must have the same order of integration as the change in the spot rate.

[5] Baillie and Bollerslev (2000) use a model where there is long memory in the variances of exchange rates.

Our analysis is instead motivated by recent work that explains apparent long memory behavior in economic time series as resulting from various types of ignored structural changes. In particular, Lobato and Savin (1998), Granger (1999), Granger and Hyung (1999), Granger and Teräsvirta (1999), and Diebold and Inoue (2001), show that long memory behavior can be easily generated from structural breaks or regime switching. Moreover, Sakoulis and Zivot (2001) have found evidence for structural changes in the mean and variance of the forward discount that appear to be linked to events like discrete changes in monetary policy and changes in interest rates due to the business cycle.

3 A Brief Review of Long Memory Process

In this section, we review some basic properties of long memory processes, and then discuss the nonparametric estimators of the long memory parameter, *d*, that are used in our empirical analysis.

3.1 Definition of Long Memory Process

We can define the long memory property of a time series in several ways. They can be based on a time domain or frequency domain model. The key point is that long memory processes are defined in terms of restrictions on the second moment, such as, the autocorrelations or the spectral density.[6]

Consider a covariance stationary process $\{Y_t\}$. In the time domain, $\{Y_t\}$ exhibits long memory if its autocorrelations $\rho(k)$ exhibit slow decay and persistence such that

$$\sum_{k=-n}^{n} |\rho(k)| \to \infty \text{ as } n \to \infty . \tag{9}$$

For a short memory covariance stationary process, the autocorrelation function is geometrically bounded

$$|\rho(k)| \leq cr^{-k}, \; k = 1,2,\ldots, \tag{10}$$

where c is a positive constant and $0 < r < 1$. Stationary long memory process have autocorrelations that satisfy hyperbolic decay such that

$$\rho(k) \sim ck^{2d-1} \text{ as } n \to \infty , \tag{11}$$

[6] For more details on the statistical properties of long memory processes, see Beran (1994), Baillie and Bollerslev (1994), and Baillie (1996).

where $c \neq 0$ and the long memory parameter has the range $0 < d < \frac{1}{2}$. If $d < 0$, $\{Y_t\}$ has "intermediate" or "anti-persistent" memory since $\sum_{-\infty}^{\infty} |\rho(k)| < \infty$.

Long memory in the frequency domain is defined when we evaluate the spectral density function at frequencies that tend to zero. Suppose the spectral density function $f(w)$ has the following property

$$f(w) \sim c|w|^{-2d} \text{ as } w \to 0^+. \tag{12}$$

Then $\{Y_t\}$ exhibits long memory where the stationary range of d is the same as in the time domain definition above.

In the empirical literature, fractionally integrated ARMA (ARFIMA) processes satisfy the above conditions. Hosking (1981), Granger and Joyeux (1980), Diebold and Rudebusch (1989), Baillie and Bollerslev (1994) provide details on these models. The process $\{Y_t\}$ is defined as an ARFIMA(*p, d, q*) process if

$$(1-L)^d \phi(L) Y_t = \theta(L)\varepsilon_t, \tag{13}$$

where $\phi(z)$ and $\theta(z)$ are autoregressive and moving average polynomials, respectively, with roots outside the unit circle and $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$. The fractional differencing lag operator $(1-L)^d$ is defined by the binomial expansion

$$\begin{aligned}(1-L)^d &= \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(k+1)\Gamma(-d)} \\ &= 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \cdots,\end{aligned} \tag{14}$$

where $\Gamma(\bullet)$ is the gamma function. From equation (14), the coefficient on the differencing lag operator provides the rate of declining weight. An ARFIMA process is said to be a stationary process when $-0.5 < d < 0.5$. However, we can divide this area into two parts. First, for $0 < d < 0.5$, the process is called stationary long memory. Second, for $-0.5 < d < 0$ it is called anti-persistent memory. When $d \geq 0.5$, the process is nonstationary but mean reverting with finite impulse response weights. When $d \geq 1$, the process is nonstationary and non-mean reverting. In this paper, our interest is not in estimating all of the parameters of an ARFIMA process but rather in estimating the long memory parameter d to evaluate the evidence for long memory in the forward discount.

3.2 Nonparametric Estimation of the Long Memory Parameter

There are several nonparametric and semiparametric estimation methods for the long memory parameter d of a fractional process. The most common are the log periodogram regression, the re-scaled range (R/S statistic) and the local Whittle estimator.

The most popular method to estimate d is the log periodogram regression estimator suggested by Geweke and Porter-Hudak (1983). To describe this estimator, let $Z_t = (1-L)Y_t$. Then we can estimate d by examining

$$(1-L)^{\tilde{d}} Z_t = \phi^{-1}(L)\theta(L)\varepsilon_t = u_t, \tilde{d} = d-1. \qquad (15)$$

The spectral density of Z_t is given by

$$f_z(w) = \left|1-e^{-iw}\right|^{-2\tilde{d}} f_u(w) = \left|2\sin(\frac{w}{2})\right|^{-2\tilde{d}} f_u(w), \qquad (16)$$

where $f_u(w)$ is the spectral density of u_t. After taking logs and adding and subtracting $\log\left[f_u(0)\right]$ from equation (16), we have the following equation (17), which we evaluate at the harmonic ordinates

$$\log\left[f_z(w_j)\right] = \log\left[f_u(0)\right] - \tilde{d}\log\left[4\sin^2\left(\frac{w_j}{2}\right)\right] + \log\left[f_u(w_j)/f_u(0)\right], \tag{17}$$

GPH propose an estimator of $\tilde{d}$ based on the first m periodogram ordinates $w_1, w_2, \ldots, w_{m,}$ that is $I_z(w_j)$ where $j = 1,2,\ldots,m$,

$$\log\left[I_z(w_j)\right] = \alpha + \beta\log\left[4\sin^2\left(\frac{w_j}{2}\right)\right] + u_j, \tag{18}$$

and $u_j = \log\left[I_z(w_j)/f_z(w_j)\right]$. From (17) and (18), the GPH estimator $\tilde{d}$ is the slope of a least squares regression; i.e., regress $\log\left[I_z(w_j)\right]$ on a constant and the explanatory variable $\log\left|4\sin^2\left(\frac{w_j}{2}\right)\right|$ in the sample $j = 1,2,\ldots,m$ giving the estimate $\hat{d}_{GPH} = -\hat{\beta}$. GPH show that with a proper choice of m, such as $m = g(T) < T$ which has the properties $\lim_{T\to\infty} g(T) = \infty$, $\lim_{T\to\infty} g(T)/T = 0$, then $\hat{d}_{GPH} \xrightarrow{p} \tilde{d}$. Furthermore, if $\lim_{T\to\infty} (\ln T)^2 / g(T) = 0$ then $\hat{d}_{GPH}$ has the limiting distribution

$$\hat{d}_{GPH} \overset{A}{\sim} N\left(\tilde{d}, \frac{\pi^2}{6\sum_{j=1}^{m}\left(U_j - \bar{U}\right)^2}\right), \tag{19}$$

where $U_j = \log\left|4\sin^2\left(\frac{w_j}{2}\right)\right|$. GPH prove consistency and asymptotic normality of $\hat{d}_{GPH}$ only for $\tilde{d} < 0$. More recently, Robinson (1990) developed a proof of consistency for $0 < d < 0.5$ and Velasco (1999) provides some asymptotic theory for $0.5 < d < 1$ under additional restrictions.

Agiakloglou, Newbold and Wohar (1993), argued that the GPH estimator $\hat{d}_{GPH}$ has serious finite sample bias and is very inefficient if u_t in (15) is an AR(1) or MA(1) process, which is relevant in the present context since the forward discount is often modeled as a highly persistent AR(1) process. They also argued that the GPH estimator is not invariant to first differencing and tests based on it may be seriously misleading.

To avoid these problems, we use the modified log periodogram (MLP) regression due to Kim and Phillips (2000) which is applicable for the nonstationary range $d \geq 0.5$, and is robust for AR(1) and MA(1) errors. Kim and Phillips argued that one usually has no prior information about the order of d before estimation, thus it is prudent to cover a wide range of plausible parameter values of d. They also showed that there are several advantages to using the MLP regression. In particular, it modifies the periodogram ordinates to find the correct form of the data generating process for the discrete Fourier transforms (DFT), which is simple and involves no unknown parameters. Additionally, consistency of the estimator for d can be obtained under weak conditions without assuming a distributional form for the errors. Finally, the estimation procedure is very easy to implement, since it is basically just a least squares regression with a transformed dependent variable.

The MLP regression to test for long memory when d is in the range $0.5 < d < 1$ is based on the levels of the data and has the form[7]

$$\log I_{vx}(w_j) = c - 2d \log\left|1 - e^{iw_j}\right| + b(w_j), \tag{20}$$

where

[7] They also provide an estimator that is valid when d is in the range $1 < d < 2$ and when $d = 1$.

$$b(w_j) = a(w_j) + \log\frac{I_{vx}(w_j;d)}{I_{vx}(w_j)} + O_p\left(\frac{1}{j^{1-d}}\right)$$

$$a(w_j) = \log\left[I_u(w_j)/f_u(0)\right], c = \log f_u(0).$$

The MLP regression estimator of d is obtained by regressing $\log I_{vx}(w_j)$ on $\log\left|1-e^{iw_j}\right|$ over frequencies $\{w_j\}$, $j = 1,2,\ldots,m$ and is given by

$$\hat{d}_{MLP} = -\frac{1}{2}\left[\sum_{j=1}^{m} x_j^2\right]^{-1}\left[\sum_{j=1}^{m} x_j \log I_{vx}\right], \tag{21}$$

where $x_j = \log\left|1-e^{iw_j}\right| - \overline{\log\left(\left|1-e^{iw}\right|\right)}$ and $\overline{\log\left(\left|1-e^{iw}\right|\right)} = \frac{1}{m}\sum_{j=1}^{m}\log\left|1-e^{iw_j}\right|$.

Kim and Phillips also consider a trimmed version of the modified LP regression that does not rely on the data being Gaussian. The trimmed estimator of d has the form

$$\hat{d}_{MLP} = -\frac{1}{2}\left[\sum_{j=l}^{m} x_j^2\right]\left[\sum_{j=l}^{m} x_j \log I_{vx}(w_j)\right], \tag{22}$$

where $j = 1,\ldots,l-1$ have been trimmed out of the regression. They showed that

$\sqrt{m}(\hat{d}_{MLP} - d) \xrightarrow{d} N\left(0, \frac{\pi^2}{24}\right)$ under mild regularity conditions.

A practical problem with the implementation of the MLP regression is the choice of m, the number of periodogram ordinates. GPH suggested using $m = T^{\alpha}$ with $\alpha = \frac{1}{2}$, where T is the sample size. Diebold and Rudebusch (1989) also used this rule in their Monte Carlo simulation results. However, Sowell (1992) argued that we should consider the shortest cycle associated with long run behavior when we choose m. More recently, Hurvich *et al.* (1998) showed that the optimal m, minimizing the asymptotic mean squared error, is of order $O(T^{4/5})$ and proved the asymptotic normality of $\hat{d}_{GPH}$ using this rule. Maynard and Phillips (2001) chose $\alpha = 0.75$ with 3,000 observations. Kim and Phillips (2000) suggested that $0.7 < \alpha < 0.8$ is desirable for their estimation method based on simulation experiments, and used $m = T^{7/10}$ and $m = T^{4/5}$ to analyze the extended Nelson-Plosser data. In this paper, we use three different values of m; $m = T^{7/10}$, $m = T^{7.5/10}$, $m = T^{8/10}$.

4 Multiple Mean Break Model

In this section, we briefly review the methodology of Bai and Perron (1998, 2003) for estimation and inference in a simple multiple mean break model that is utilized in our empirical analysis.

Bai (1997a, b) and Bai and Perron (1998, 2003), hereafter BP, consider several methods for the estimation of single and multiple structural breaks in dynamic linear regression models. They estimate the unknown break points given T observations by the least squares principle, and provide general consistency and asymptotic distribution results under fairly weak conditions allowing for serial correlation and heteroskedasticity.[8]

We consider the simple structural change in mean model, because structural breaks in the mean have a natural interpretation as the direct effect of an economic shock to the forward discount/interest rate differential. The pure structural change model we consider is defined as

$$y_t = c_j + u_t,\ t = T_{j-1}+1,\ldots,T_j \tag{23}$$

for $j = 1,\ldots,m+1$, $T_0 = 0$, $T_{m+1} = T$, and $y_t = f_t - s_t$[9]. The process is subject to m breaks and c_j is the mean of the forward discount for each regime. The model allows for serial correlation in the errors and heterogeneity of the residuals across segments.

[8] Hidalgo and Robinson (1996) suggest a test for structural change when a structural break point is known in a long memory environment. They, however, provide the test when the stochastic process is Gaussian. In practice, we do not know the structural break point and it is also difficult to tell whether the process is Gaussian.

[9] Our model is different from that used by Sakoulis and Zivot (2001). Sakoulis and Zivot (2001) use the partial structural change model as follows

$$f_t - s_t = c_j + \phi(f_{t-1} - s_{t-1}) + u_t,\ t = T_{j-1}+1,\ldots,T_j$$

for $j = 1,\ldots,m+1$, $T_0 = 0$ and $T_{m+1} = T$. ϕ is the AR coefficient of the lagged forward discount, which is not subject to structural change. We find similar break dates with the partial break model as Sakoulis and Zivot's result.

We can estimate the mean change model as follows. For each m-partition $(T_1,\ldots T_m)$ we obtain the least squares estimate c_j by minimizing the sum of squared residuals

$$S_T(T_1,\ldots,T_m)=\sum_{j=1}^{m+1}\sum_{t=T_{j-1}+1}^{T_j}\left(y_t-c_j\right)^2 \tag{24}$$

giving $\hat{c}_j(\{T_1,\ldots T_m\})$ as the mean estimates associated with the given m-partition that minimizes $S_T(T_1,\ldots,T_m)$. The estimated break points are defined by

$$\left(\hat{T}_1,\ldots\hat{T}_m\right)=\operatorname{argmin}_{T_1,\ldots T_m} S_T(T_1,\ldots T_m), \tag{25}$$

where the minimization occurs over all possible m-partitions. Using the estimated break points $\left(\hat{T}_1,\ldots\hat{T}_m\right)$ we find the regression parameter estimates $\hat{c}_j(\{T_1,\ldots,T_m\})$.

BP show that the break fractions $\hat{\lambda}_i=\frac{\hat{T}_i}{T}$ converge to their true value λ_i^0 at rate T under very general conditions, but that the estimated break dates $\hat{T}_i$ are not consistent. They propose a method to construct approximate confidence intervals for the break dates based on a novel asymptotic theory that assumes the magnitudes of the breaks (mean shifts in our model) decrease as the sample size increases.

Let $SupF_T(l)$ be the F statistic of no structural breaks $(l=0)$ versus $m=k$ breaks when the break dates are unknown, and let L denote the maximum number of breaks allowed. BP suggest the use of two statistics to determine if structural change has occurred: the double maximum statistic, $UD\max=\max_{1\le l\le L}\sup F_T(l)$; and the weighted double max statistic $UD\max=\max_{1\le l\le L} w_l\cdot\sup F_T(l)$, where the weights to the individuals $SupF_T(l)$ tests are such that marginal p-values are equal across values of l. The null hypothesis of both tests is no structural breaks against the alternative of an unknown number of breaks given some specific upper bound L. They also recommend using the $SupF_T(l+1\,|\,l)$ test statistics, which test the null of l breaks versus the alternative of $l+1$ breaks. Critical values for these tests can be found in Bai and Perron (1998, 2003).

BP suggest the following strategy to determine if structural change has occurred, and, if structural change has occurred, to determine the number of breaks. First use the $UD\max$ and $WD\max$ tests to see if at least one break is present. If there is evidence for structural change, select the number of structural breaks using the sequential $SupF_T(l+1|l)$ statistics starting with $l = 1$. This procedure may be complemented with selecting the number of breaks by minimizing a Bayesian Information Criterion (BIC) or a modified Schwarz' Criterion (LWZ) due to Liu, Wu and Zidek (1997). Based on a small set of Monte Carlo experiments, they find the sequential procedure to be generally more reliable than the model selection criteria.

5 Empirical Results

5.1 Data

We consider the same data as Sakoulis and Zivot (2001), which is monthly exchange rate data in terms of US dollars for five G7 countries: Germany, France, Italy, Canada, and Great Britain. All rates are end-of-month, average of bid and ask rates, and span the period 1976:1-1999:1. The Japanese Yen is not considered since the sample period is different (i.e., from 1978:7 to 1999:1). We multiplied the natural log of all rates by 100, so that the differences in rates are in percentages.

Table 1 reports summary statistics for the exchange rate series. Except for the Canadian Dollar, all forward discounts have right skewed distributions. The Italian Lira and French Franc have long right tails compared to the others. Also, the Italian Lira and French Franc's distributions are peaked relative to normal in terms of kurtosis. The Jarque-Bera test shows that all forward discount rates reject the normal distribution hypothesis.

To summarize the time series properties of the forward discount, we first report unit root tests of the forward discount for each country in Table 2. We use the ADF-GLS t-statistic of Elliot, Rothenberg and Stock (1996), which is more powerful than the ADF t-statistic for highly persistent alternatives. We estimate the ADF-GLS test that includes a constant in the test regression. Following Ng and Perron (2001), the lag length of the test regression was chosen by the modified AIC with the maximum number of lag length of 15. The ADF-GLS unit root test results in Table 2 provide mixed results for the order of integration of the forward discount for each country. We fail to reject the null hypothesis of a unit root in the forward discount for German mark, Canadian dollar and British Pound. Theses results are similar to the KPSS test results of Crowder (1994). On the other hand, we reject the null hypothesis for the French Franc and Italian Lira.

5.2 Estimation of the Long Memory Parameter Before Adjusting for Structural Breaks

Table 3 reports $\hat{d}_{MLP}$ for each series for three different values of the periodogram ordinates *m*. All estimates are greater than 0.5, but less than 1, with reasonably small estimated standard errors. We conclude that if we do not consider structural breaks, all forward discounts have non-stationary long memory components. However the series have mean reverting properties with infinite variance, but finite cumulative impulse response weights.

The results for Germany are quite similar to those of Baillie and Bollerslev (1994).[10] They reported that the German Mark is *I*(0.77), and our estimates of the order of integration for *m* = 0.70, 075, and 0.80 are in the range $\hat{d}_{MLP} \in (0.618, 0.725)$. Canada has $\hat{d}_{MLP} \in (0.619, 0.695)$, which is slightly greater than Baillie and Bollerslev's estimate of *I*(0.45). The U.K. has a higher value of *d* than other forward discounts, $\hat{d}_{MLP} \in (0.786, 0.866)$, and is also greater than those of Baillie and Bollerslev's *I*(0.55). The French Franc has $\hat{d}_{MLP} \in (0.589, 0.680)$ and the Italian Lira has $\hat{d}_{MLP} \in (0.536, 0.648)$, which are lower than the estimates for the other countries, and correspond with our ADF-GLS unit root test results. From these results, we find that the U.K. has the most non-stationary series whereas Italy has the most stationary. Moreover, the only 95% confidence interval for *d* that includes the non-stationary and non-mean reverting range $d \geq 1$ is for the U.K. with $m = n^{0.8}$.

[10] They use the monthly forward discount for the Canadian dollar forward discount, the German mark forward discount and the British pound forward discount from January 1974 to December 1991.

5.3 The Number of Breaks in the Forward Discount

Results for the mean break model based on the BP methodology are reported in Tables 4 through 8.[11] For the German Mark, the $UD\max$ and $WD\max$ tests provide evidence of multiple structural breaks. $SupF_T(1)$ is not significant but $SupF_T(2)$ through $SupF_T(5)$ are all significant at the 5% level. Also, $SupF_T(l+1|1)$ is significant at the 1% level when l=4 which suggests 5 breaks. The BIC and LWZ also choose 5 breaks. Thus, we choose 5 breaks for the German Mark. Note that the fourth structural change date has a broad confidence interval but the other break dates are precisely estimated.[12] In the case of the French Franc, the $UD\max$, and $WD\max$ tests also indicate the presence of multiple breaks. The sequential procedure, BIC and LWZ all suggest 4 breaks. For the Italian Lira the $UD\max$ and $WD\max$ tests suggest the presence of multiple structural changes, but the $SupF_T(l+1\,|\,l)$ statistics are not significant $l \geq 1$ which indicates only 1 break. In contrast, the LWZ suggests 2 breaks and the BIC suggests 4 breaks. Additionally, $SupF_T(l)$ test does reject the null of no breaks versus one, two, three and four. Therefore, we choose 4 breaks for Italian Lira. For the Canadian Dollar, the $UD\max$ and $WD\max$ tests are significant at 1% level. $SupF_T(3|2)$ is significant at the 1% level and the BIC and LWZ choose 3 breaks, so we choose 3 breaks for the Canadian Dollar.[13] In the case of the British Pound, the $UD\max$ and $WD\max$ tests are significant at 1% level, $SupF_T(5|4)$ is significant at the 1% level, the BIC suggests 5 breaks and the LWZ suggests 4 breaks. We choose 5 breaks for the British Pound.

Similar to Sakoulis and Zivot (2001), most of the structural break dates estimated for each country are very similar to the break dates for the rest of the countries. For all the countries, almost

[11] We allow up to 5 breaks and used a trimming $\varepsilon = 0.05$ which implies that each segment has at least 13 observations.
[12] Also note that Kanas (1998) finds evidence for up to six breaks in ERM exchange rates.
[13] BP suggest that we don't need to impose similar restrictions of $SupF_T(k)$ on the number of breaks for different values of ε.

half of the breaks occur during the beginning of the sample. From Figure 1, we see that during the period 1980-83 the forward discounts are very volatile and the behavior is consistent with the change in the U.S. Central Bank's policy objective, as well as the subsequent recession of 1981-82.

To check the robustness of our estimated structural change dates to the base currency used to define the exchange rates, we re-estimate the structural break models for the forward discount using the British Pound as the base currency. These results are summarized in Tables 9 through 13 and Figure 2 We use the same methods to choose the number of structural breaks as described above. In general, for most countries the break dates in the forward discount using the British pound as the base currency are similar to the breaks found using the U.S. dollar as the base currency.[14] We choose 4 breaks for the German Mark, and 3 of these break dates are close to break dates found using the U.S. dollar as the base currency. We find 5 break dates for the French Franc, and, except the first break date, these breaks are very similar to those found using the U.S. dollar as the base currency. We choose 3 break dates for Italian Lira, and these dates are only slightly different than those in terms of U.S. dollar. The number of structural breaks in the Canadian dollar is same as the number found using the U.S. dollar. However, as shown in Figure 2, the behavior of the forward discount of the Canadian dollar in terms of the British pound is quite different than the forward discount in terms of U.S. dollar, and the location of structural break dates are different.

[14] We estimate the long memory parameter using the MLP regression with and without structural breaks in terms of British Pound. The results are available upon request.

5.4 Tests for Long Memory Parameter after Adjusting for Structural Breaks

Table 14 reports $\hat{d}_{MLP}$ for each series after adjustment for the estimated structural breaks. That is, the table shows estimates of d using the residual series $\hat{u}_t = y_t - \hat{c}_j$. Note that all estimates of d are lower using the residual series.

A potential criticism of our results is that the estimated break dates are potentially spurious if the data is in fact nonstationary. However, Granger and Hyung (1999) show that for simulated nonstationary data after allowing for structural breaks the estimated parameter d only provides evidence of possible spurious break points when it is less than zero. Our estimates of d after removing structural breaks from the original series are all positive, and all of the 95% confidence intervals for d exclude values of $d < 0$.

5.5 Comparison of Autocorrelation Functions

Figures 3 through 7 show each country's autocorrelations of the spot exchange rate and forward discount before and after adjusting for structural breaks for each country. From the definition of long memory, the autocorrelations of a long memory process should lie between the autocorrelations of a stationary autoregressive process and a non-stationary process.

Consider the results for the German Mark. The autocorrelations of the spot exchange rate have a typical non-stationary shape. The autocorrelations of the forward discount are slightly less than the spot exchange rate, but both have similar paths. The autocorrelations of the forward discount after adjusting for the estimated structural breaks exhibit exponential decay more typical of stationary data. Now consider the autocorrelations for the French Franc. The autocorrelations of the forward discount show a degree of persistence that is considerably less than those of the spot exchange rates. The autocorrelations of the break-adjusted forward discount also exhibit a more rapidly decaying pattern, but not as quickly as that of the German Mark. The autocorrelations of the forward discounts for the Italian Lira and Canadian dollar are very similar to those for French Franc, and the autocorrelations of the break-adjusted forward discount for the British Pound are similar to the German Mark.

6. Conclusion

In this paper, we have analyzed the long memory properties of the monthly forward discount series for five G7 countries with and without allowing for structural breaks in the mean. We used the MLP regression to nonparametrically estimate the long memory parameter of a data series, since it is preferred to the GPH estimator when the data may have a nonstationary component. Also, we used the Bai and Perron method to detect and estimate the number of breaks in the mean. We found that multiple breaks in the mean are present and that most break dates are associated with periods of high volatility in the beginning of the sample period. We found that when we allow for structural breaks, the forward discounts' persistence is considerably less than before adjusting for structural breaks. This result is consistent with Granger and Hyung (1999)'s and Diebold and Inoue (2001)'s arguments. However, we may not conclude that the long memory properties are totally due to structural breaks since we found evidence of long memory in the forward discount after allowing for structural breaks. One significant difference is that when we allow for structural breaks, the estimates change from implying nonstationary long memory with mean reversion to stationary long memory. From the autocorrelations, we found that once these breaks are allowed for, the autocorrelations are drastically reduced.

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Figure 1: Estimated Break Dates for Forward Discount (/ U.S. Dollar)

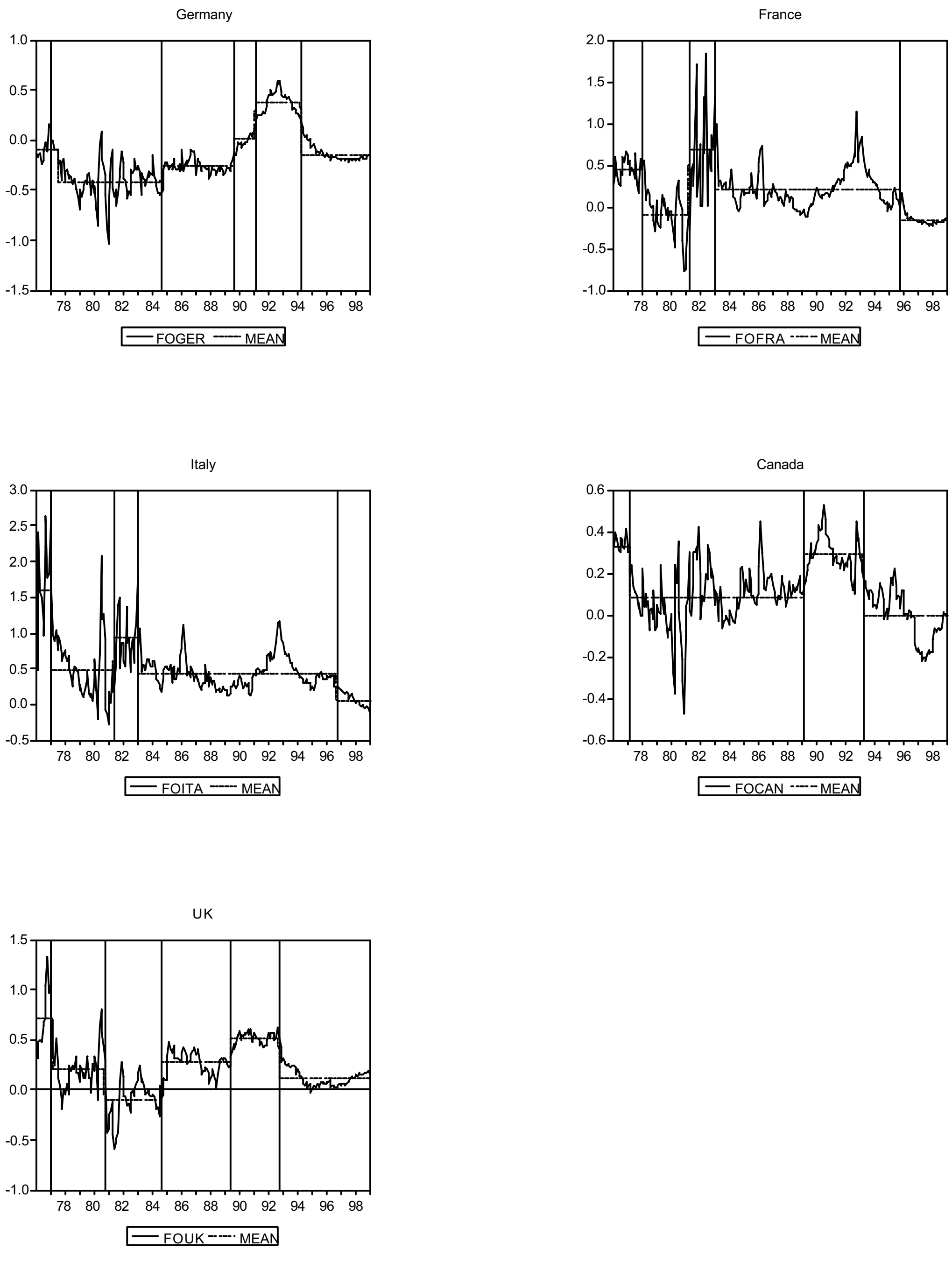


Figure 2: Estimated Break Dates for Forward Discount (/ British Pound)

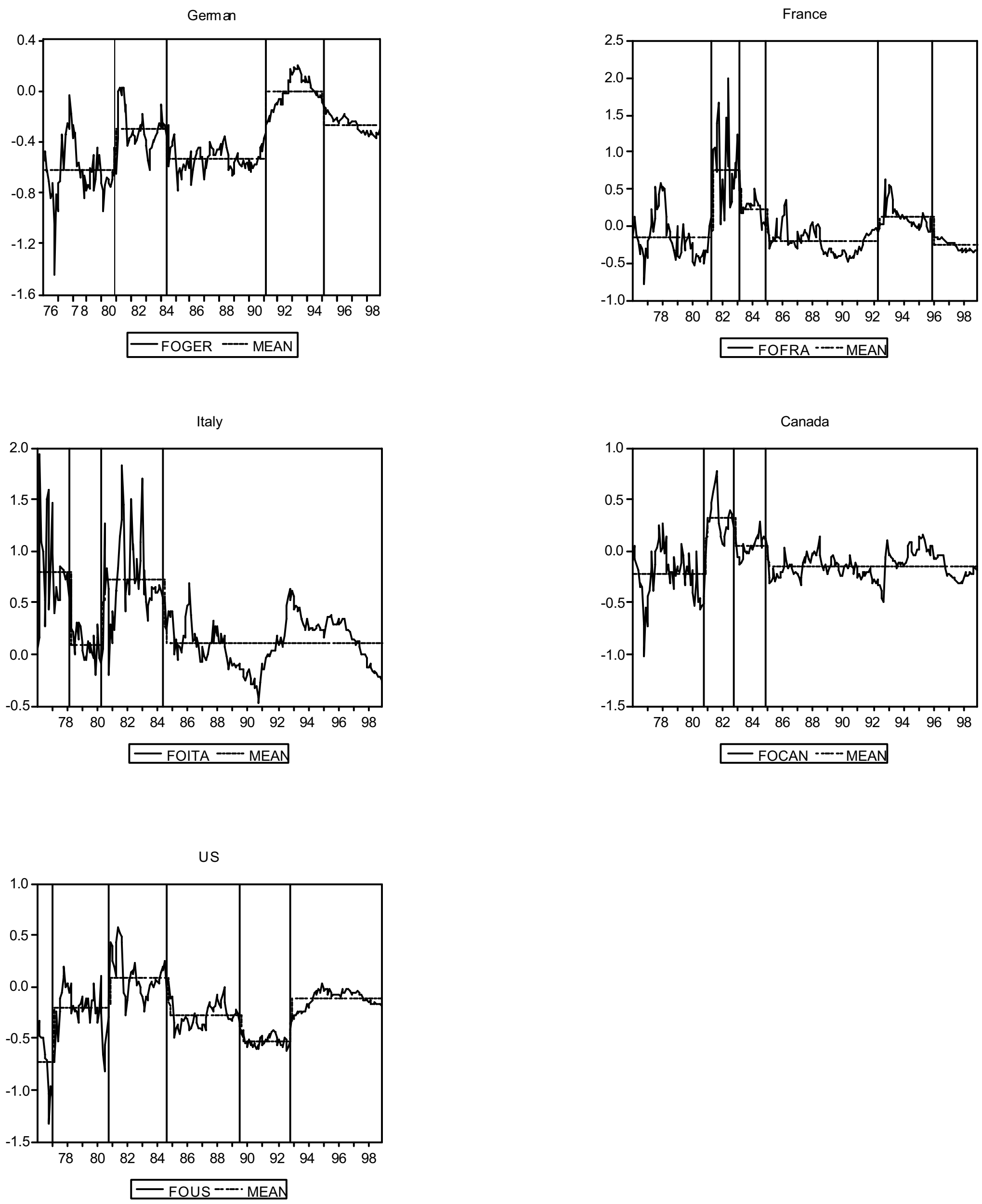
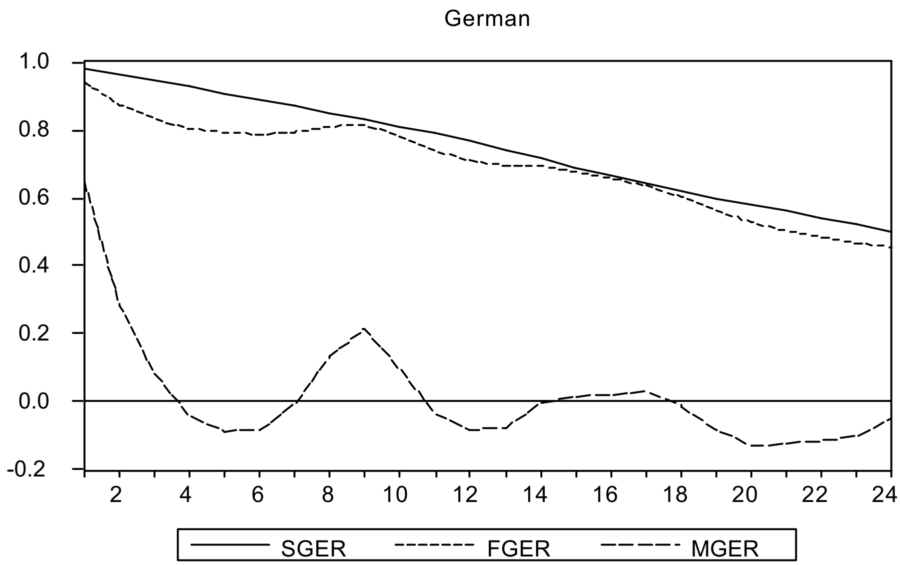
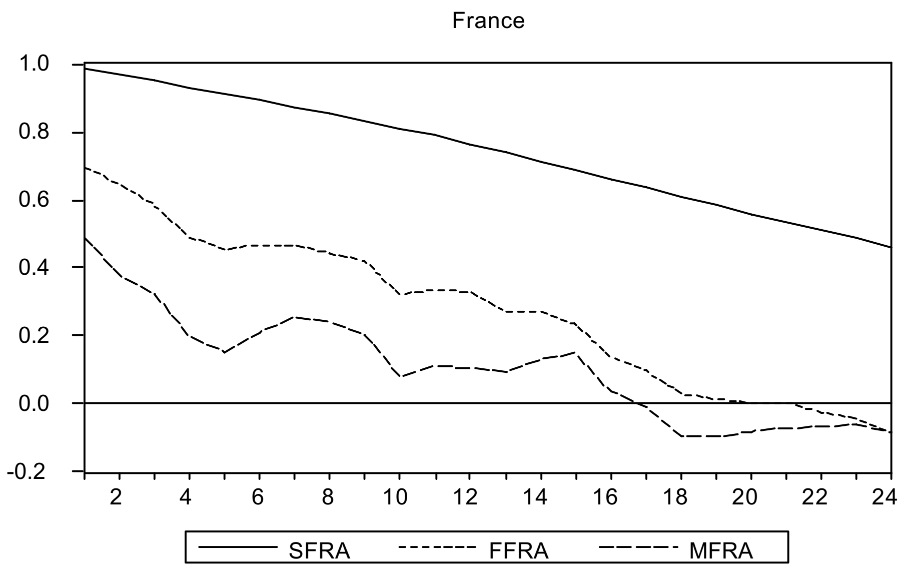


Figure 3: Autocorrelation Function for German Mark



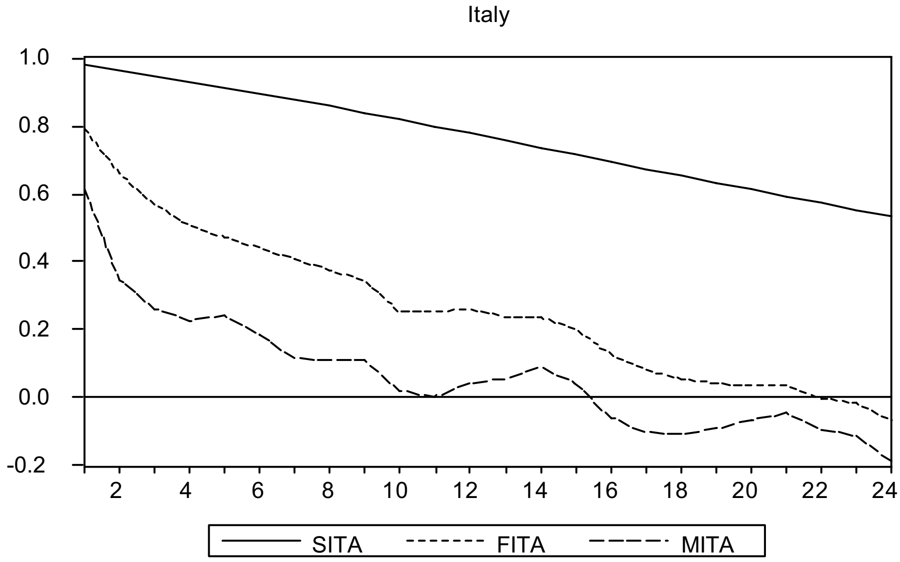
* SGER: The spot exchange rate for German Mark, FGER: The forward discount for German Mark, and MGER : The mean adjusted forward discount for German Mark.

Figure 4: Autocorrelation Function for French Franc



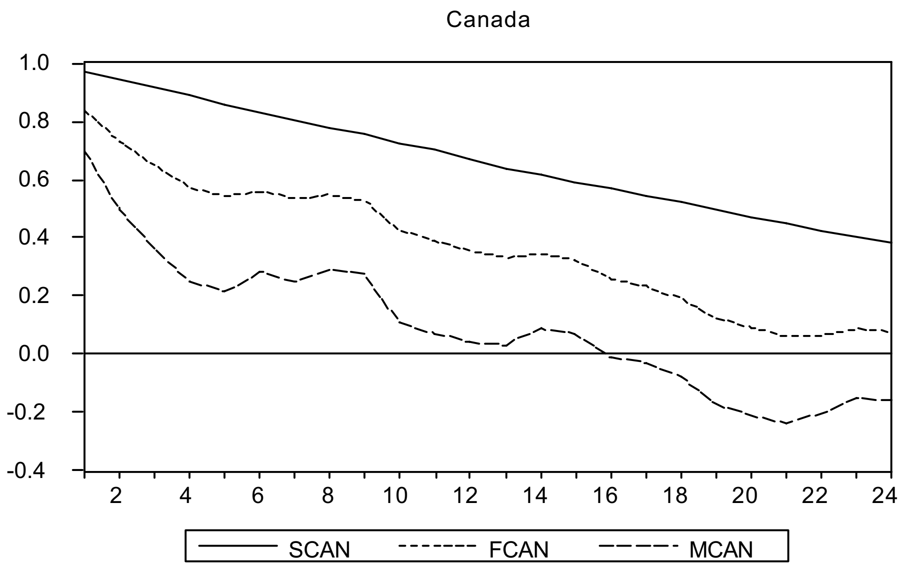
* SFRA: The spot exchange rate for French Franc, FFRA: The forward discount for French Franc, and MGER : The mean adjusted forward discount for French Franc.

Figure 5: Autocorrelation Function for Italian Lira



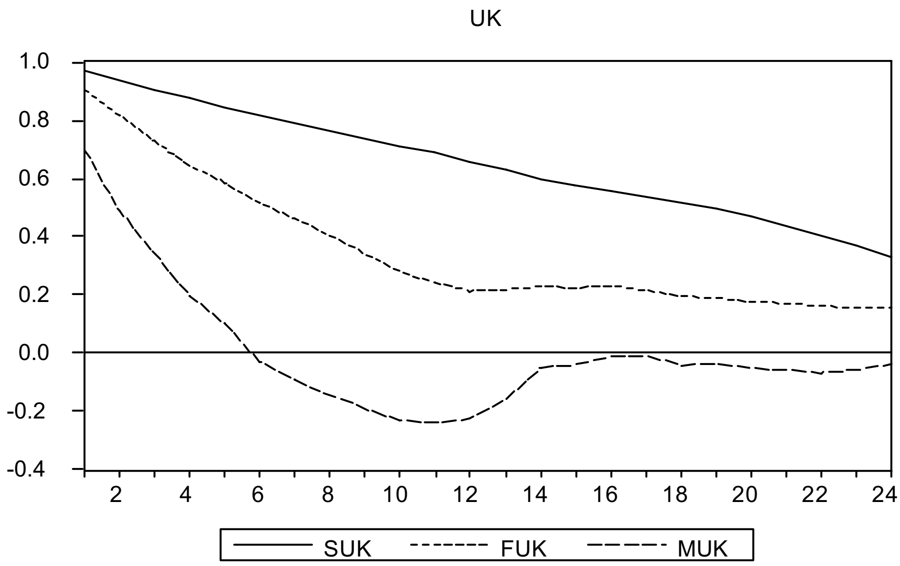
* SITA: The spot exchange rate for Italian Lira, FGER: The forward discount for Italian Lira, and MGER : The mean adjusted forward discount for Italian Lira

Figure 6: Autocorrelation Function for Canadian Dollar



* SCAN: The spot exchange rate for Canadian Dollar, FCAN: The forward discount for Canadian Dollar, and MCAN : The mean adjusted forward discount for Canadian Dollar.

Figure 7: Autocorrelation Function for British Pound



* SUK: The spot exchange rate for British Pound, FUK: The forward discount for British Pound and MUK: The mean adjusted forward discount for British Pound.

Table 1: Summary Statistics

	German Mark	*French Franc*	*Italian Lira*	*Canadian Dollar*	*British Pound*
Mean	-0.163	0.176	0.500	0.113	0.215
S.D.	0.279	0.331	0.432	0.162	0.260
Skewness	0.588	1.203	2.052	-0.208	0.323
Kurtosis	3.630	7.119	9.027	3.277	4.653
Jarque-Bera Test	20.520	261.760	611.510	2.875	36.280

Table 2: Unit Root Test

	ADF-GLS Test	
	Test Statistics	Lag Length
German Mark	-1.548	9
French Franc	-2.112*	11
Italian Lira	-4.412**	11
Canadian Dollar	-1.179	10
British Pound	-1.766	12

1) *, ** denote significant at the 5% and 1% respectively.
2) 5% and 1% critical values are -1.941 and -2.573 respectively.
3) The ADF-GLS test lag was selected by the Modified AIC rule.

Table 3: Estimates of MLP Regression for Before Structural Break (/ U.S. Dollar)

	m	$\hat{d}$	S.D.	95% C.I.
German Mark	$m = n^{0.7}$	0.725	0.101	[0.525, 0.924]
	$m = n^{0.75}$	0.618	0.086	[0.448, 0.789]
	$m = n^{0.8}$	0.707	0.074	[0.562, 0.852]
French Franc	$m = n^{0.7}$	0.615	0.101	[0.415, 0.814]
	$m = n^{0.75}$	0.680	0.086	[0.510, 0.851]
	$m = n^{0.8}$	0.589	0.074	[0.444, 0.734]
Italian Lira	$m = n^{0.7}$	0.536	0.101	[0.337, 0.736]
	$m = n^{0.75}$	0.589	0.086	[0.419, 0.760]
	$m = n^{0.8}$	0.648	0.074	[0.503, 0.793]
Canadian Dollar	$m = n^{0.7}$	0.619	0.101	[0.419, 0.818]
	$m = n^{0.75}$	0.652	0.086	[0.481, 0.822]
	$m = n^{0.8}$	0.695	0.074	[0.550, 0.840]
British Pound	$m = n^{0.7}$	0.786	0.101	[0.586, 0.985]
	$m = n^{0.75}$	0.841	0.086	[0.792, 0.877]
	$m = n^{0.8}$	0.866	0.074	[0.721, 1.011]

Table 4: The Pure Structural Break Estimation Results: German Mark / U.S. Dollar

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 5$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
0.536	14.657***	10.868***	16.223***	12.882***	16.233***	25.919***
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
19.819***	15.580***	17.234***	18.235***			
			Number of Breaks Selected			
Sequential	0					
LWZ	5					
BIC	5					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	-0.084(0.026)	$\hat{T}_1$	1977:08 [1976:02 1978:01]			
$\hat{c}_2$	-0.412(0.013)	$\hat{T}_2$	1984:09 [1984:06 1988:12]			
$\hat{c}_3$	-0.249(0.015)	$\hat{T}_3$	1989:09 [1988:08 1989:10]			
$\hat{c}_4$	0.022(0.029)	$\hat{T}_4$	1991:02 [1977:09 1991:08]			
$\hat{c}_5$	0.375(0.019)	$\hat{T}_5$	1994:04 [1994:03 1994:05]			
$\hat{c}_6$	-0.141(0.016)					

1) *, **, *** indicate 10%, 5% and 1% significance respectively.

2) 95% confidence interval for break dates

Table 5: The Pure Structural Break Estimation Results: French Franc / U.S. Dollar

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 5$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
88.187^{***}	23.609^{***}	24.320^{***}	83.774^{***}	-	88.187^{***}	133.842^{***}
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
17.865^{***}	77.715^{***}	67.242^{***}	-			
			Number of Breaks Selected			
Sequential	4					
LWZ	4					
BIC	4					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	0.443(0.046)	$\hat{T}_1$	1978:02 [1976:10 1978:03]			
$\hat{c}_2$	-0.104(0.038)	$\hat{T}_2$	1981:04 [1980:11 1981:12]			
$\hat{c}_3$	0.682(0.050)	$\hat{T}_3$	1983:02 [1980:10 1983:11]			
$\hat{c}_4$	0.211(0.019)	$\hat{T}_4$	1995:10 [1995:09 1995:11]			
$\hat{c}_5$	-0.157(0.038)					

1) $^{*}, ^{**}, ^{***}$ indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates

Table 6: The Pure Structural Break Estimation Results: Italian Lira / U.S. Dollar

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 5$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
35.253^{***}	15.102^{***}	11.994^{***}	12.193^{***}	-	32.523^{***}	32.523^{***}
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
0.072	1.987	3.820	-			
			Number of Breaks Selected			
Sequential	1					
LWZ	4					
BIC	2					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	1.597(0.081)	$\hat{T}_1$	1977:02 [1976:04 1977:07]			
$\hat{c}_2$	0.474(0.042)	$\hat{T}_2$	1981:06 [1980:12 1985:10]			
$\hat{c}_3$	0.930(0.067)	$\hat{T}_3$	1983:02 [1980:03 1983:07]			
$\hat{c}_4$	0.434(0.023)	$\hat{T}_4$	1996:09 [1980:10 1998:07]$^{+}$			
$\hat{c}_5$	0.059(0.058)					

1) $^{*}, ^{**}, ^{***}$ indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates
3) $^{+}$ indicates 90% C.I.

Table 7: The Pure Structural Break Estimation Results: Canadian Dollar / U.S. Dollar

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 5$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
7.354	13.635***	22.249***	-	-	22.249***	32.246***
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
58.365***	47.625***	-	-			
			Number of Breaks Selected			
Sequential	0					
LWZ	3					
BIC	3					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	0.327(0.029)	$\hat{T}_1$	1977:04 [1976:03 1977:05]$^+$			
$\hat{c}_2$	0.085(0.009)	$\hat{T}_2$	1989:03 [1988:02 1991:01]			
$\hat{c}_3$	0.292(0.017)	$\hat{T}_3$	1993:04 [1990:04 1993:08]			
$\hat{c}_4$	-0.006(0.014)					

1) *, **, *** indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates
3) + indicates 90% C.I.

Table 8: The Pure Structural Break Estimation Results: British Pound / U.S. Dollar

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 5$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
8.200*	5.283	7.541	8.216	43.238***	43.238***	74.799***
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
2.681	12.873**	32.941***	32.941***			
			Number of Breaks Selected			
Sequential	1					
LWZ	5					
BIC	5					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	0.719(0.039)	$\hat{T}_1$	1977:02 [1976:06 1978:09]			
$\hat{c}_2$	0.202(0.022)	$\hat{T}_2$	1980:10 [1978:04 1981:09]			
$\hat{c}_3$	-0.096(0.021)	$\hat{T}_3$	1984:09 [1983:11 1986:10]			
$\hat{c}_4$	0.277(0.019)	$\hat{T}_4$	1989:06 [1989:04 1991:06]			
$\hat{c}_5$	0.518(0.023)	$\hat{T}_5$	1992:10 [1992:05 1992:11]			
$\hat{c}_6$	0.115(0.017)					

1) *, **, *** indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates

Table 9: The Pure Structural Break Estimation Results: German Mark / British Pound

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 4$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
5.474	29.530***	21.890***	29.331***	-	29.530***	46.860***
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
19.380***	66.807***	66.807***	-			
			Number of Breaks Selected			
Sequential	0					
LWZ	4					
BIC	4					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	-0.622 (0.018)	$\hat{T}_1$	1980:12 [1979:08 1982:12]			
$\hat{c}_2$	-0.294 (0.022)	$\hat{T}_2$	1984:06 [1983:08 1987:03]			
$\hat{c}_3$	-0.528 (0.015)	$\hat{T}_3$	1991:03 [1990:08 1991:05]			
$\hat{c}_4$	0.000 (0.020)	$\hat{T}_4$	1995:02 [1994:09 1997:01]			
$\hat{c}_5$	-0.262 (0.021)					

1) *, **, *** indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates

Table 10: The Pure Structural Break Estimation Results: French Franc / British Pound

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 5$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
4.685	20.920***	26.039***	20.690***	21.133***	26.039***	37.739***
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
20.729***	14.923***	2.633***	14.923**			
			Number of Breaks Selected			
Sequential	0					
LWZ	5					
BIC	5					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	-0.149 (0.030)	$\hat{T}_1$	1981:04 [1980:11 1982:01]			
$\hat{c}_2$	0.762 (0.052)	$\hat{T}_2$	1983:02 [1982:10 1984:03]			
$\hat{c}_3$	0.212 (0.052)	$\hat{T}_3$	1984:12 [1983:03 1985:04]			
$\hat{c}_4$	-0.206 (0.026)	$\hat{T}_4$	1992:04 [1991:06 1995:02]			
$\hat{c}_5$	-0.115 (0.036)	$\hat{T}_5$	1995:12 [1995:06 1996:09]			
$\hat{c}_6$	-0.268 (0.041)					

1) *, **, *** indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates

Table 11: The Pure Structural Break Estimation Results: Italian Lira / British Pound

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 3$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
13.063^{**}	11.993^{***}	30.119^{***}	-	-	30.119^{***}	43.651^{***}
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
3.576	25.945^{***}	-	-			
			Number of Breaks Selected			
Sequential	1					
LWZ	3					
BIC	3					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	0.793 (0.054)	$\hat{T}_1$	1978:03 [1978:01 1978:08]			
$\hat{c}_2$	0.083 (0.055)	$\hat{T}_2$	1980:05 [1979:09 1980:06]			
$\hat{c}_3$	0.728 (0.040)	$\hat{T}_3$	1984:06 [1980:01 1985:03]			
$\hat{c}_4$	0.113 (0.021)					

1) *, **, *** indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates

Table 12: The Pure Structural Break Estimation Results: Canadian Dollar / British Pound

			Specification			
$Z_t = \{1\}$	$q = 1$	$p = 0$	$h = 13$	$M = 3$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
2.671	4.613	9.206^{**}	-	-	9.206^{*}	13.343^{**}
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
9.889^{**}	15.104^{***}	-	-			
			Number of Breaks Selected			
Sequential	0					
LWZ	3					
BIC	3					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	-0.223 (0.020)	$\hat{T}_1$	1980:11 [1978:11 1981:07]			
$\hat{c}_2$	0.333 (0.033)	$\hat{T}_2$	1982:10 [1982:10 1985:02]			
$\hat{c}_3$	0.046 (0.031)	$\hat{T}_3$	1984:12 [1979:09 1985:06]			
$\hat{c}_4$	-0.142 (0.011)					

1) *, **, *** indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates

Table 13: The Pure Structural Break Estimation Results: U.S Dollar / British Pound

			Specification			
$Z_t = \{1\}$	$q=1$	$p=0$	$h=13$	$M=5$		
			Tests			
$SupF_T(1)$	$SupF_T(2)$	$SupF_T(3)$	$SupF_T(4)$	$SupF_T(5)$	$UD\max$	$WD\max$
8.185*	5.311	7.529	8.205	43.289***	43.289***	74.888***
$SupF(2\|1)$	$SupF(3\|2)$	$SupF(4\|3)$	$SupF(5\|4)$			
2.814	12.849**	42.817***	42.817***			
			Number of Breaks Selected			
Sequential	1					
LWZ	5					
BIC	5					
			Break Point Dates (with Confidence Interval)			
$\hat{c}_1$	-0.719(0.039)	$\hat{T}_1$	1977:02 [1976:06 1978:09]			
$\hat{c}_2$	-0.202(0.022)	$\hat{T}_2$	1980:10 [1978:04 1981:09]			
$\hat{c}_3$	0.096(0.021)	$\hat{T}_3$	1984:09 [1983:11 1986:10]			
$\hat{c}_4$	-0.277(0.019)	$\hat{T}_4$	1989:06 [1989:04 1991:06]			
$\hat{c}_5$	-0.518(0.023)	$\hat{T}_5$	1992:10 [1992:05 1992:11]			
$\hat{c}_6$	-0.115(0.017)					

1) *, **, *** indicate 10%, 5% and 1% significance respectively.
2) 95% confidence interval for break dates

Table 14: Estimates of MLP Regression for After Structural Break (/ U.S. Dollar)

	M	$\hat{d}$	S.D.	95% C.I.
German Mark	$m = n^{0.7}$	0.229	0.101	[0.029, 0.429]
	$m = n^{0.75}$	0.254	0.086	[0.084, 0.425]
	$m = n^{0.8}$	0.369	0.074	[0.224, 0.514]
French Franc	$m = n^{0.7}$	0.295	0.101	[0.095, 0.495]
	$m = n^{0.75}$	0.378	0.086	[0.207, 0.548]
	$m = n^{0.8}$	0.323	0.074	[0.177, 0.468]
Italian Lira	$m = n^{0.7}$	0.356	0.101	[0.157, 0.556]
	$m = n^{0.75}$	0.339	0.086	[0.169, 0.510]
	$m = n^{0.8}$	0.426	0.074	[0.281, 0.572]
Canadian Dollar	$m = n^{0.7}$	0.450	0.101	[0.251, 0.650]
	$m = n^{0.75}$	0.517	0.086	[0.347, 0.688]
	$m = n^{0.8}$	0.582	0.074	[0.436, 0.727]
British Pound	$m = n^{0.7}$	0.303	0.101	[0.103, 0.502]
	$m = n^{0.75}$	0.369	0.086	[0.199, 0.540]
	$m = n^{0.8}$	0.455	0.074	[0.310, 0.600]