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Gaetano Lisi

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**EERI**
**Economics and Econometrics Research Institute**
Avenue de Beaulieu
1160 Brussels
Belgium

Tel: +322 298 8491
Fax: +322 298 8490
www.eeri.eu

# Matching models and housing markets: the role of the zero-profit condition

GAETANO LISI
Creativity and Motivations (CreaM) Economic Research Centre
Department of Economics and Law
University of Cassino and Southern Lazio, Italy
E-mail: gaetano.lisi@unicas.it

***Abstract***

*The recent and growing literature which has extended the use of search and matching models even to the housing market does not use the free entry or zero-profit assumption as a key condition for solving the equilibrium of the model. This is because a straightforward adaptation of the basic matching model to the housing market seems impossible. However, this paper shows that the zero-profit condition can be easily reformulated to take the distinctive features of the housing market into account. Indeed, it helps to provide a theoretical explanation for well-known empirical regularities in the housing markets.*

## 1. INTRODUCTION

Recently, there has been much focus on formulising the behaviour of the housing market through the search and matching models usually used for the labour market (see, among others, Diaz and Jerez, 2009; Novy-Marx, 2009; Piazzesi and Schneider, 2009; Genesove and Han, 2010; Leung and Zhang, 2011; Peterson, 2012). The housing market is in fact a "matching market" like the labour market, that clears not only through price but also through time and money that the parties spend on the market. Thus, the search and matching approach is certainly also suitable for this type of market.

However, all of these models do not use the free entry or zero-profit assumption as a key condition to solve the equilibrium of the model. Important differences between the labour and the housing markets seem to make a straightforward adaptation of the basic matching model (see Pissarides, 2000) to the study of the real estate market impossible. In particular, the free entry or zero-profit condition for job creation in the labour market seems to have no counterpart in the housing market. In fact, the assumption that vacancies in the housing market are created until the asset value of a vacant house is equal to zero may make sense if houses are supplied perfectly elastically by competitive house builders, in addition to being supplied by owners who no longer need them for occupation.

Nevertheless, the zero-profit condition can be easily reformulated to take the distinctive feature of the housing market into account, where buyers today are potential sellers tomorrow (Leung, Leong and Wong, 2006), and without introducing the construction sector interpretation. Precisely, the zero-profit condition can be used to find the number of sellers and buyers in the housing market equilibrium. In short, since the value of a vacant house is merely the value of being a seller, the zero-profit condition allows us to obtain the equilibrium of the model where the transition process from seller (buyer) to buyer (seller) comes to an end. When the value of being a seller is equal to zero, in fact, no one will be willing to become a seller and thus the matching no longer occurs. In equilibrium, in fact, all the profit opportunities derived from buying/selling houses have been exploited.

As will be clear later, the developed model (where the zero-profit condition plays a key role) is able to provide a theoretical explanation for well-known empirical regularities in the housing markets.

The rest of the paper is organised as follows: section 2 presents a simple model of the housing market where the zero-profit condition leads to equilibrium; section 3 extends the model by studying in detail the transition process from seller (buyer) to buyer (seller), while section 4 concludes the work.

## 2. A MATCHING MODEL OF HOMEOWNERSHIP MARKET

To make our point as simply as possible, we consider the model developed by Lisi (2011, 2012).[1] In the model, at a point in time, there is a mass of sellers (*s*) and a mass of buyers (*b*), and the population of buyers and sellers is normalised to the unit, i.e. $1 = s + b$ .[2]

This matching model focuses on the transition process from seller (buyer) to buyer (seller), thus taking the homeownership market into account. Sellers are assumed to hold $h \geq 2$ houses of which $h-1$ are on the market. It follows that vacancies ($v$) are simply given by $v = (h-1) \cdot s$ . In this way, it is therefore possible for a buyer to become a seller and vice versa. In fact, when a seller (with two houses) manages to sell one house, s/he becomes a buyer, and then when s/he increases his/her inventory of houses to two by buying in the market, s/he becomes a seller again. Furthermore, in the homeownership market if a contract is legally binding, it is no longer possible to return to the circumstances preceding the bill of sale, unless a new and distinct contractual relationship is set up. Therefore, the destruction rate of a specific buyer-seller match does not exist and the value of an occupied home for a seller is simply given by the selling price.

[1] Indeed, the quoted model (Lisi, 2011, 2012) does not provide sufficient insights to understand the role of the zero-profit condition and thus the free-entry assumption does not seem to suit the housing market. Also, the model predicts the existence of different types of buyers and sellers; nevertheless, it does not study in detail the transition process from seller to buyer and vice versa.
[2] There can certainly be households who are neither buyers nor sellers at a given point in time, but this case does not change the main results of the analysis.

As a result, the expected values of a vacant house ($V$) and of buying a house ($H$) are given by:[3]

$$rV = -c + q(\vartheta)\cdot[P - V] \tag{1}$$

$$rH = -e + g(\vartheta)\cdot[x - H - P] \tag{2}$$

where $\vartheta \equiv v/b$ is the housing market tightness which identifies the market frictions which prevent (or delay) the matching between the parties (standard hypothesis of constant returns to scale in the matching function, $m = m(v,b)$, is adopted and standard technical assumptions are postulated: $q(\vartheta) = m(v,b)/v$ and $g(\vartheta) = m(v,b)/b$ are, respectively, the instantaneous probability of filling a vacant house and of buying a home, with $q'(\vartheta) < 0$, $g'(\vartheta) > 0$, and $lim_{\vartheta\to 0}q(\vartheta) = lim_{\vartheta\to\infty}g(\vartheta) = \infty$, $lim_{\vartheta\to 0}g(\vartheta) = lim_{\vartheta\to\infty}q(\vartheta) = 0$). The terms $c$ and $e$ represent, respectively, the cost flows sustained by sellers and buyers during the search. When a match takes place, the risk neutral buyer gets a linear benefit $x$ from the property and pays the sale price $P$ to the seller.

First of all, note that the value of a vacant house is nothing but the value of being a seller. Also, vacancies depend on the number of sellers, which in turn depends on market tightness,

$$\vartheta \equiv v/b = \frac{(h-1)\cdot s}{1-s} \Rightarrow s = \frac{\vartheta}{\vartheta + (h-1)} \tag{3}$$

hence, the zero-profit condition can be reformulated in this housing market matching model to find the share of sellers and buyers in equilibrium. Eventually, in fact, the person stops trying to buy further as s/he is either a seller or a buyer, but not both, at any point in time. Indeed, the transition process from seller (buyer) to buyer (seller) comes to an end when the value of being a seller is equal to zero. In this case, in fact, no one will be willing to become a seller and thus the matching no longer occurs. Formally,

$$V = 0 \Rightarrow \frac{c}{q(\vartheta)} = P \Rightarrow \vartheta^* \Rightarrow s^*, b^* \Rightarrow v^* \tag{4}$$

[3] Time is continuous and individuals are risk neutral, live infinitely and discount the future at the exogenous interest rate $r > 0$.

In equilibrium, in fact, all the profit opportunities derived from buying/selling houses have been exploited. It follows that, given the equilibrium value of market tightness, we find the (total) share of sellers, buyers and also the vacancy rate.

The zero-profit condition (4) gives a positive relationship between selling price and market tightness: an increase in $P$ increases $\vartheta$, since the (expected) time-on-the-market, i.e. the inverse of the probability of filling a vacancy $\frac{1}{q(\vartheta)}$, is increasing in $\vartheta$.

The sale price $P$ is instead obtained by the *Nash bargaining solution* usually used for decentralised markets:

$$P = argmax\left\{(P-V)^{\gamma} \cdot (x-H-P)^{1-\gamma}\right\} \Rightarrow P = \gamma \cdot (x-H) \tag{5}$$

where $0 < \gamma < 1$ is the share of the bargaining power of sellers. Note that the selling price depends negatively on market tightness (the *congestion externalities* effect on the sellers' side), since $H$ is increasing in $\vartheta$.

Only one long-term equilibrium with a positive value of $P$ and $\vartheta$ exists in the model. This testable proposition is made possible by a model with a positive relationship between housing prices and time on the market (which starts from the origin of the axes) and a downward sloping price function (with positive vertical intercept).

The model is able to reproduce the observed joint behaviour of prices and time-on-the-market: in fact, the house with a higher price has a longer time on the market; whereas, the longer the time-on-the-market the lower the sale price (Leung, Leong, and Chan, 2002). Furthermore, different bargaining strengths and search costs can explain price dispersion in housing prices (Leung, Leong and Wong, 2006): in fact, in this case, housing prices would be different even for similar houses (i.e. houses which give the same benefit *x*). Therefore, the model succeeds in providing a theoretical explanation for well-known empirical regularities.

In conclusion, although the model used is rather general and simple (the next section, however, tries to enrich it), the zero-profit condition has a clear economic meaning since it is used to find the equilibrium of the model where the transition process from seller (buyer) to buyer (seller) comes to an end. More important, the zero-profit condition helps to reproduce stylised facts such as the relationship between time-on-the-market and selling price and the empirical anomaly known as price dispersion in a straightforward manner.

## 3. THE TRANSITION PROCESS

This section extends the model to study the transition process in detail, thus taking into account the different kinds of buyers and sellers. Although homelessness seems equivalent to unemployment, changes in the housing markets are very different from shifts in the labour market (Wheaton, 1990). Hence, in the housing market is more interesting to study the transition from seller to buyer (and vice versa) rather than the dynamic in and out of the homelessness.

As regards the sellers who hold more than two houses, the dynamics are very simple:

$$\dot{s}(h=5)=[s(h=6)-s(h=5)]\cdot q(\vartheta)\Rightarrow s(h=6)=s(h=5)$$

$$\dot{s}(h=4)=[s(h=5)-s(h=4)]\cdot q(\vartheta)\Rightarrow s(h=5)=s(h=4)$$

$$\dot{s}(h=3)=[s(h=4)-s(h=3)]\cdot q(\vartheta)\Rightarrow s(h=4)=s(h=3)$$

Thus generalising, $\Rightarrow s(h)=s(h+1),\ \forall\, h>2$ .

Obviously, more interesting is the dynamics of threshold values, namely the dynamics of buyers with one-house and sellers who hold two houses:

$$\dot{b}(h=1)=s(h=2)\cdot q(\vartheta)-[b(h=1)-b(h=0)]\cdot g(\vartheta)$$

$$\Rightarrow s(h=2)=\frac{[b(h=1)-b(h=0)]\cdot g(\vartheta)}{q(\vartheta)} \tag{6}$$

$$\dot{s}(h=2)=b(h=1)\cdot g(\vartheta)-[s(h=2)-s(h=3)]\cdot q(\vartheta)$$

$$\Rightarrow b(h=1)=\frac{[s(h=2)-s(h=3)]\ \cdot q(\vartheta)}{g(\vartheta)} \tag{7}$$

A positive share of buyers (with one-house) and sellers (who hold two houses) requires that $b(h=1)>b(h=0)$ and $s(h=2)>s(h=3)$, which are realistic conditions, i.e. a limited number of homeless people and a limited number of owners with many homes. Note that we can write the total share of sellers and buyers as follows:

$$s^* = s(h=2) + s(h=3) + s(h=4) + \dots$$

$$\Rightarrow s^* = s(h=2) + n \cdot s(h=3) \tag{8}$$

$$b^* = b(h=1) + b(h=0) \tag{9}$$

Hence, we get a system of four equations (6 – 8) in four unknowns: $b(h=1)$, $b(h=0)$, $s(h=2)$, and $s(h=3)$. By reducing it to a system of two equations in two unknowns, $b(h=0)$ and $s(h=2)$, we obtain:

$$b(h=0) = \frac{b^*}{2} - \frac{s(h=2)}{2} \cdot \frac{q(\vartheta)}{g(\vartheta)} \tag{$i$}$$

$$b(h=0) = b^* - \frac{[s(h=2) \cdot (n+1)/n - s^*/n] \cdot q(\vartheta)}{g(\vartheta)} \tag{$ii$}$$

It is straightforward to find that:

- $b^* + \frac{[s^*/n] \cdot q(\vartheta)}{g(\vartheta)} > \frac{b^*}{2}$, i.e. the vertical intercept of function (*ii*) is higher than the vertical intercept of function (*i*);
- The slope of the function (*ii*) $\left( -\frac{[(n+1)/n] \cdot q(\vartheta)}{g(\vartheta)} \right)$ is steeper than the slope of the function (*i*) $\left( -\frac{1}{2} \cdot \frac{q(\vartheta)}{g(\vartheta)} \right)$.

Hence, an equilibrium with positive values of $b(h=0)$ and $s(h=2)$ can not be ruled out *ex-ante* and it depends on the values of $\vartheta$ and *n*.

However, a more simple way to close the model is to assume that there are no homeless buyers, i.e. $b(h=0)=0$, and then we no longer need the dynamics equation for $b(h=1)$, since $b(h=1)=b^*$. Therefore, we get a system of two equations in two unknowns:

$$\dot{s}(h=2) = 0 \Rightarrow b^* \cdot \frac{g(\vartheta)}{q(\vartheta)} + s(h=3) = s(h=2) \tag{I}$$

$$\Rightarrow s^* - n \cdot s(h=3) = s(h=2) \tag{II}$$

Thus obtaining,

$$b^* \cdot \frac{g(\vartheta)}{q(\vartheta)} + s(h=3) = s^* - n \cdot s(h=3) \Rightarrow s(h=3) = \frac{1}{n+1} \cdot \left( s^* - b^* \cdot \frac{g(\vartheta)}{q(\vartheta)} \right)$$

$$\Rightarrow s(h=2) = s^* - \frac{n}{n+1} \cdot \left( s^* - b^* \cdot \frac{g(\vartheta)}{q(\vartheta)} \right)$$

Hence, sufficient condition for the existence of an interior equilibrium is that $s^* - b^* \cdot \frac{g(\vartheta)}{q(\vartheta)} > 0$, i.e. $q(\vartheta)$ is sufficiently higher than $g(\vartheta)$, i.e. the perspective of becoming seller is very attractive. Indeed, in the model the purchase is directed to the sale of the property.

## 4. Conclusions

Almost all the matching models of the housing market do not use the free entry or zero-profit assumption as a key condition to solve the equilibrium of the model. This is because a straightforward adaptation of the basic matching model to the housing market seems impossible. In this paper we show that the zero-profit condition can be easily reformulated to take the distinctive features of the housing market into account. Precisely, the zero-profit condition is used to find the equilibrium of the model where the transition process from seller (buyer) to buyer (seller) comes to an end. Furthermore, the zero-profit condition helps to reproduce stylised facts of housing markets, such as the relationship between time-on-the-market and selling price and the empirical anomaly known as price dispersion, in a straightforward way. Finally, the model is extended in order to study the transition process from seller (buyer) to buyer (seller) in detail.

## References


Díaz, Antonia, and Belén Jerez (2009), House Prices, Sales and Time on the Market: A Search-Theoretic Framework, Working Paper 09-25 Economic Series, Departamento de Economía, Universidad Carlos III de Madrid.

Genesove, David, and Lu Han (2010), Search and Matching in the Housing Markets, CEPR Discussion Papers, 7777.

Leung, Charles Ka Yui, and Zhang, Jun (2011), Fire Sales in Housing Market: Is the House–Search Process Similar to a Theme Park Visit ? *International Real Estate Review*, 14(3), 311-329.

Leung, Charles Ka Yui, Youngman Chun Fai Leong, and Siu Kei Wong (2006), "Housing Price Dispersion: An Empirical Investigation", *The Journal of Real Estate Finance and Economics*, 32(3), 357-385.

Leung, Charles Ka Yui, Youngman Chun Fai Leong, and Ida Yin Sze Chan (2002), Time-On-the-Market: Why Isn't Price Enough? *International Real Estate Review*, 5(1), 91-115.

Lisi, Gaetano (2011). The Hedonic Price Function in a Matching Model of Housing Market. *Economics Bulletin*, 31(3), 2347-2355.

Lisi, Gaetano (2012). On the Functional Form of the Hedonic Price Function: A Matching-theoretic Model and Empirical Evidence. *International Real Estate Review*, Forthcoming.

Novy-Marx, Robert (2009), Hot and Cold Markets, *Real Estate Economics*, 37(1), 1-22.

Peterson, Brian M. (2012), Fooled by Search: Housing Prices, Turnover and Bubbles, Bank of Canada Working Paper 2012-3.

Piazzesi, Monika, and Martin Schneider (2009), Momentum Traders in the Housing Market: Survey Evidence and a Search Model, *American Economic Review: Papers & Proceedings*, 99(2), 406-411.

Pissarides, Christopher A. (2000), *Equilibrium Unemployment Theory* (2nd edition), MIT Press Books.

Wheaton, William C. (1990), "Vacancy, Search, and Prices in a Housing Market Matching Model", *Journal of Political Economy*, 98(6), 1270-1292.