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Progressive taxation and (in)stability in an exogenous growth model with non-market (”home”) production

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Abstract

We show that in a exogenous growth model with non-market (”home”) sector calibrated to Bulgarian data under the progressive taxation regime (1993-2007), the economy exhibits equilibrium indeterminacy due to the presence of non-market production. These results are in line with the findings in Benhabib and Farmer (1994, 1996) and Farmer (1999). Also, the findings in this paper are in contrast to Guo and Lansing (1988) who argue that progressive taxation works as an automatic stabilizer. Under the flat tax regime (2008-16), the economy calibrated to Bulgarian data displays saddle-path stability. The decrease in the average effective tax rate addresses the indeterminacy issue and eliminates the ”stable focus” dynamics.

Keywords: Progressive taxation; Non-market Sector; Home Production; Equilibrium (In)determinacy

JEL Classification: H22, J46, D51, D91, O41

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1 Introduction and Motivation

Tax policies, and in particular personal income taxation policies, are known to affect households incentives to invest in physical capital, and their decisions to provide labor services to businesses. The analysis of the effect of tax policies within the framework of exogenous growth models with a representative agent is relatively recent, *e.g.*, King and Rebelo (1990). This paper adds to earlier research by focusing on the market vs. non-market ("home") sector labor choice, and the home production technology is viewed as an alternative (labor-intensive) way to produce goods and services. It has to choose how much to work in each sector, where the two types of consumption, market and non-marker, are modelled as imperfect substitutes. The presence of the home production sector, and the sectoral labor supply decision margin creates interesting interactions in the model, as shown also in Vasilev (2015b) in a setup with grey economy.

As in Chen and Guo (2015) and Vasilev (2016), the focus in this paper is to examines the instability effect of progressive taxation in the case of Bulgaria pre-2008 and compare and contrast the results to the flat tax reform regime in place as of 2008. Importantly, our work differs from that earlier study. While our findings are qualitatively similar to that in Chen and Guo (2013, 2015), here there is no endogenous growth, and the mechanism is based on labor allocation between the market and non-market sector. By investment in physical capital, the after-tax marginal productivity of labor is kept from decreasing, as compared to the return to labor in the market sector. Earnings from the non-market sector are not taxed, though, which creates a sector-specific externality, which as pointed out in Farmer (1999), could create indeterminacy.

Our results come in stark contrast to Guo and Lansing (1988) who argue that a sufficiently progressive tax schedule can stabilize a real-business-cycle (RBC) model, which possesses an indeterminate steady-state against fluctuations driven by animal spirits. Indeed, in standard Keynesian setups, progressivity of the tax system is regarded as an automatic stabilizer. This is no longer the case in our model with non-market sector. The reason is that since output estimates generally impute the size of the home production sector, but income taxes are levied on official market production only, non-market sector produces increasing returns to scale.[1] The theoretical setup used in this paper to study the flat tax reform in Bulgaria is a standard RBC model with home production as in Benhabib *et al.* (1991) and MacGrattan *et al.* (1997), and augments their framework with a sufficiently-detailed government sector to capture the distortionary effect of personal income taxation in Bulgaria. From early 1990s, up until Dec. 31, 2007, Bulgaria applied progressive income taxation on personal income,[2] with tax brackets for 2007 reported in Table 1 below.

Table 1: Progressive Income Taxation in Bulgaria in 2007

Monthly taxable income (in BGN)	Tax owed
0-200	Zero-bracket amount
200-250	20% on the amount earned above BGN 200
250-600	BGN 10 + 22% on the excess over BGN 250
> 600	BGN 87 + 24% on the excess over BGN 600

Source: author's calculations.

As of January 1, 2008, a proportional (flat) tax rate of 10% on personal income was introduced. To compensate workers at the bottom of the income distribution, who suddenly faced a positive tax rate, the monthly minimum wage was increased: it went up in several steps eventually reaching BGN 420 as of Jan. 2016. Overall, under proportional taxation system featuring a lower effective income tax rate than the corresponding rate under the progressive regime, a significant reallocation of labor from unregistered activities to the official sector was observed (Vasilev 2015b). This relocation was driven by the increase to after-tax return to labor in the registered market economy, and thus making working in the non-market sector less attractive. In addition, since labor and capital are complements in the production of market output at the aggregate level, the increase in official employment increases the marginal productivity of capital. In turn, the higher return to physical capital provides a strong incentive for households to increase capital accumulation, thus enhancing the productive capacity of the economy. This generates a saddle-path dynamics by decreasing

[1] This is easily established using the specific functional forms for official and unofficial production provided later in the paper. For a similar model with grey economy, see Vasilev (2017).

[2] The description of the progressive tax system in Bulgaria in this section follows the structure used in Vasilev (2017).

the magnitude of the IRS due to the shrinking of the size of the non-market ("home") output.

The rest of the paper is organized as follows: Section 2 presents the model setup and defines the equilibrium system. Section 3 describes the data used and the calibration procedure. Section 4 characterizes the model economy's long-run behavior under both the progressive and proportional income taxation regimes. Section 5 evaluates the model stability around the steady-state for both the progressive taxation and flat-tax regimes. Section 6 discusses the results, and Section 7 concludes.

2 Model Setup

2.1 Household

The model is very similar in spirit to Vasilev (2017): There is a representative agent ("one-member household"), who is infinitely-lived, and maximizes utility out of composite consumption and leisure:

$$U = \sum_{t=0}^{\infty} \beta^t \Big\{ \ln(c_{mt} + \theta c_{nt}) + \gamma \ln(1 - h_{mt} - h_{nt}) \Big\}, \tag{2.1}$$

where

$$y_{nt} = c_{nt} = A_{nt} h_{nt}^{1-\alpha} \tag{2.2}$$

is home, or non-market, production (and consumption), c_{mt} denotes market consumption, $0 < \beta < 1$ is the discount factor, h_{mt} and h_{nt} are hours worked in the market and home sector, respectively. A_{nt} is the level of technology in period t, and labor intensity in the non-market sector equals $1-\alpha$. Parameter $\gamma > 0$ measures the relative weight that the household attaches to leisure (time off work) versus composite consumption. The degree of imperfect substitutability between market and home consumption is measured by parameter $0 < \theta < 1$.

The hourly wage rate in the market sector is w_{mt}, so total labor income in the market sector is $w_{mt}h_{mt}$. Similarly, the implicit hourly wage rate in the non-market sector is denoted by w_{nt}, so total labor income in the market sector equals $w_{nt}h_{nt}$. In addition to the labor income generated, each household saves by investing i_t in physical capital. As an owner of capital, the household receives gross interest income $r_t k_t$ from renting the capital to the firms; r_t is the before-tax return to private capital, and k_t denotes physical capital stock in the beginning of period t. Each household's physical capital evolves according to the following law of motion:

$$k_{t+1} = i_t + (1 - \delta)k_t, \tag{2.3}$$

where $0 < \delta < 1$ is the depreciation rate on capital.

Finally, the household owns the firm in the market sector, and receive all profit (π_t) in the form of dividends. The household's aggregate budget constraint is

$$(1 + \tau^c)c_t + k_{t+1} - (1 - \delta)k_t = (1 - \tau(y_{mt}))[w_{mt}h_{mt} + r_t h_{mt}] + w_{nt}h_{nt} + \pi_t + g_t^t, \tag{2.4}$$

where τ^c is the consumption tax rate, g_t^t are lump-sum government transfers, and, as in Guo and Lansing (1988), the progressive (market) income tax rate is

$$\tau(y_{mt}) = \eta \left(\frac{y_{mt}}{y_m} \right)^{\phi} \tag{2.5}$$

where τ_t denotes the tax rate on total (capital and labor) registered income, *i.e*, $y_{mt} = r_t k_t^h + w_{mt}h_{mt}$, and y_m is the steady-state level of household's market income. In addition, $0 < \eta < 1$ and $0 \leq \phi < 1$, where ϕ measures the progressivity of the tax system, and η is the average effective tax rate in steady state.

The households acts competitively by taking prices $\{w_{mt}, w_{nt}, r_t\}_{t=0}^{\infty}$, consumption tax $\{\tau^c\}$, income tax schedule $\{\tau_t\}_{t=0}^{\infty}$ as given, and chooses allocations $\{c_t, k_{t+1}, h_{mt}, h_{nt}\}_{t=0}^{\infty}$ to maximize Eq.(2.1) s.t Eqs.(2.2)-(2.5), and the initial condition $\{k_0\}$ for physical capital stock. The optimality conditions from the household's problem, together with the transversality condition (TVC) for physical capital are as follows:

$$
\begin{aligned}
c_{mt} &: \quad \frac{1}{c_{mt} + \theta A_{nt} h_{nt}^{1-\alpha}} = \lambda_t (1 + \tau^c) && (2.6) \\
h_{mt} &: \quad \frac{\gamma}{1 - h_{mt} - h_{nt}} = \lambda_t [1 - (1+\phi)\tau_t] w_{mt} && (2.7) \\
h_{nt} &: \quad \frac{\theta(1-\alpha) A_{nt} h_{nt}^{-\alpha}}{c_{mt} + \theta A_{nt} h_{nt}^{1-\alpha}} = \frac{\gamma}{1 - h_{mt} - h_{nt}} + \lambda_t w_{nt} && (2.8) \\
k_{t+1} &: \quad \lambda_t = \beta \lambda_{t+1} \Big[1 + [1 - (1+\phi)\tau_{t+1}] r_{t+1} - \delta \Big] && (2.9) \\
TVC &: \quad \lim_{k \to \infty} \lambda_t k_{t+1} = 0, && (2.10)
\end{aligned}
$$

where λ_t is the Lagrangian multiplier of the budget constraint at time t. In Eq. (2.6), the household consumes at a point where marginal utility from market consumption equals the marginal cost imposed on the budget. Market hours in Eq. (2.7) is chosen so that the the net return from working an extra hour in the market sector equals the net cost of doing so. From Eq. (2.8), hours in the non-market sector will be chosen so that the disutility of home production at the margin equals the return to labor in the non-market sector. Eq. (2.9) describes the optimal capital stock allocations chosen in any two contiguous periods. The last expression, Eq. (2.10), is the boundary condition imposed on capital.

2.2 Stand-in Firm: market sector

There is also a representative firm in the market sector. It produces a homogeneous final product using a production function that requires physical capital k_t and labor h_{mt}. The production function is as follows:

$$y_{mt} = A_{mt} k_t^{\alpha} h_{mt}^{1-\alpha}, \tag{2.11}$$

where y_{mt} denotes market output produced in period t, A_{mt} measures the level of total factor productivity in period t, and $0 < \alpha < 1$ denote the productivity of physical capital while $1 - \alpha$ captures the productivity of labor.

The representative firm acts competitively by taking prices $\{w_{mt}, r_t\}_{t=0}^{\infty}$, and chooses $k_t, h_{mt}, \forall t$ to maximize firm's static profit:

$$\pi_t = A_{mt} k_t^{\alpha} h_{mt}^{1-\alpha} - r_t k_t - w_{mt} h_{mt} \tag{2.12}$$

In equilibrium profit is zero in all periods. In addition, market labor and capital receive their marginal products, *i.e.*

$$h_{mt} \quad : \quad w_{mt} = (1-\alpha)\frac{y_{mt}}{h_{mt}} \tag{2.13}$$

$$k_t \quad : \quad r_t = \alpha\frac{y_{mt}}{k_t} \tag{2.14}$$

2.3 Non-market (home) production

The household also has access to a technology ("home production") that uses only labor, given by $y_{nt} = A_{nt}h_{nt}^{1-\alpha}$, and A_{nt} is the level of total factor productivity of the home technology at time t. The home production results in the production of a non-market consumption good, which is an imperfect substitute for the market consumption good. As in Conesa *et al.* (2001), the labor intensive specification for the production process in the non-market sector seems to be an adequate approximation to reality. The household will optimally work h_{nt} hours in every period to maximize static profit

$$\max_{h_{nt}} A_{nt}h_{nt}^{1-\alpha} - w_{nt}h_{nt}. \tag{2.15}$$

With free entry, there are zero profits, hence the implicit wage in the home sector equals

$$w_{nt} = A_{nt}h_{nt}^{-\alpha}. \tag{2.16}$$

2.4 Government

The government collects tax revenue from market consumption, registered labor and capital income to finance government expenditure, which are then spent on wasteful government consumption $\{g_t^c\}_{t=0}^{\infty}$ and lump-sum transfers $\{g_t^t\}_{t=0}^{\infty}$. The government budget constraint is then

$$\tau^c c_{mt} + \tau(y_{mt})[r_t k_t + w_{mt}h_{mt}] = g_t^c + g_t^t. \tag{2.17}$$

Government takes prices $\{w_{mt}, r_t\}_{t=0}^{\infty}$ and allocations $\{c_{mt}, k_t, h_{mt}\}_{t=0}^{\infty}$ as given. Government consumption share in output will be set equal to its data average, so the level of $\{g_t^c\}_{t=0}^{\infty}$ will vary with output. The income tax schedule $\{\tau_t\}_{t=0}^{\infty}$ will also vary with income, while government transfers $\{g_t^t\}_{t=0}^{\infty}$ will be residually determined: it will adjust to ensure the government budget constraint is balanced in every time period.

2.5 Decentralized Competitive Equilibrium

Given the initial conditions for the state variable k_0, a Decentralized Competitive Equilibrium (DCE) is defined to be a sequence of prices $\{r_t, w_{mt}, w_{nt}\}_{t=0}^{\infty}$, allocations $\{c_{mt}, k_{t+1}, h_{mt}, h_{nt}, g_t^c, g_t^t\}_{t=0}^{\infty}$, consumption tax rate $\{\tau^c\}$, income tax schedule $\{\tau_t\}_{t=0}^{\infty}$ such that (i) household's utility is maximized; (ii) the stand-in firm in the market sector maximizes profit every period; (iii) wage rate in the non-market sector is such that profits are zero every period; (iv) government budget is balanced in each time period; (iv) all markets clear.[3]

3 Data and model calibration

The model is calibrated to Bulgarian data at annual frequency. The period under investigation is 1993-2016 where 1993-2007 is when taxation was progressive, and 2008-16 is the flat tax regime. Data on the output, household consumption, private fixed investment shares in output, employment rate, the average wage rate, and the minimum wage rate was obtained from the National Statistical Institute (NSI). Table 2 on the next page summarizes the values of all model parameters. The values were obtained following a standard approach adopted in quantitative macroeconomics. Physical capital income share is set to its average value $\alpha = 0.429$, and the labor income share is $1 - \alpha = 0.571$. Consumption tax rate is set to its rate in data, $\tau^c = 0.200$. Next, we use Vasilev's (2015b) estimate that $\delta = 0.047$, and that $K/Y = 3.491$. Next, we compute the average effective tax rate $\eta = 0.14$ and the (gross) degree of progressivity was computed to be $1 + \phi = 1.43$ for the progressive regime, and $\eta = 0.11, \phi = 0$ for the flat tax. Next, from the steady-state Euler equation, we can calibrate the discount factor $\beta = 0.969$. The relative weight on leisure in the household's utility function, parameter γ, will be set to match the steady-state share of hours in Bulgaria over the period $h_m + h_n = 0.333$. We assume that the household will work equally in the two sectors, or $h_m = h_n = 0.167$. Technology in the non-market sector is assumed to be such that workers working full time in the grey economy would earn the minimum wage. Thus parameter A_n will be set to match the ratio between market and home output in total. Normalizing steady-state market output to unity, we obtain $A_m = 1.627$ and $A_n = 0.814$.

[3] The system of equations is provided in the Appendix

Table 2: Model Parameters

Param.	Value	Definition	Source
β	0.969	Discount factor	Calibrated
α	0.429	Capital income share	Data avg.
$1-\alpha$	0.571	Labor income share	Calibrated
θ	0.250	Degree of substitutability between consumptions	Data avg.
δ	0.047	Depreciation rate of physical capital	Estimated
γ	1.631	Relative weight on leisure in utility function	Calibrated
τ^c	0.200	Consumption tax rate	Data avg.
η	{0.11; 0.14}	Average effective income tax rate (flat *vs.* progr.)	Data avg.
ϕ	{0; 0.43}	Progressivity parameter (flat *vs.* progr.)	Data avg.
A_m	1.627	Steady-state level of total factor productivity, market	Calibrated
A_n	0.814	Steady-state level of total factor productivity, home	Calibrated

4 Steady-State

Once model parameters were obtained, the steady-state ratios for the model calibrated to Bulgarian data were obtained. The results are reported in Table 3 below for both tax regimes. In particular, keeping discount factor and depreciation rate constant, a lower effective tax

Table 3: Data Averages and Long-run solution

	Description	Data	Model (progr.)	Model (flat tax)
c/y^m	Consumption-to-output ratio	0.674	0.685	0.685
i/y^m	Fixed investment-to-output ratio	0.201	0.175	0.205
g^c/y^m	Gov't consumption-to-output ratio	0.176	0.140	0.110
k/y^m	Physical capital-to-output ratio	3.491	3.491	4.115
$w^m h_m/y^m$	Labor share in output	0.571	0.571	0.571
rk/y^m	Capital share in output	0.429	0.429	0.429
h^m	Share of time spent working in the market	0.167	0.167	0.167
h^n	Share of time spent working at home	0.167	0.167	0.167

rate and no progressivity will raise the after-tax real interest rate. In turn, that would increase capital stock, and lower the hours used in the home production, and relocate that labor toward the market sector, and ultimately increase aggregate consumption.

5 Stability of Equilibrium Dynamics

The equilibrium system is now log-linearized around its unique steady-state, and after simplification, it can be represented by a system of two first-order difference equations in market consumption and physical capital:

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_{m,t+1} \end{bmatrix} = \begin{bmatrix} B1 & B2 \\ B3 & B4 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_{mt} \end{bmatrix} \tag{5.1}$$

where scalars B1, B2, B3, B4 are functions of model parameters. The characteristic roots (eigenvalues) of the dynamic system are as follows:

$$\lambda_{1,2} = \frac{(B1 + B4) \pm \sqrt{(B1 + B4)^2 - 4.(B1.B4 - B2.B3)}}{2} \tag{5.2}$$

For Bulgaria under the progressive taxation regime (1993-2007), we obtain the following values:

$$B_1 = 1.0742, B_2 = -0.6013, B_3 = 1.0209, B_4 = 0.0415$$
$$\lambda_1 = 0.5579 + 0.5893i, \lambda_2 = 0.5579 - 0.5893i$$

Given that the reduced-form representation of the equilibrium system features two characteristic roots that are complex conjugates that have real parts that are less than unity, the model features global stability (indeterminacy, or "stable focus"). Intuitively, this means that the Bulgarian economy under the progressive taxation regime can reach the steady state with highly volatile consumption. As in Farmer (1999), the non-market sector generates a sector-specific externality, as home production is not taxed. In addition, non-market output adds to total production in the computation of gross domestic product and thus the framework creates increasing returns to scale.

In contrast, for Bulgaria under the proportional (flat) tax regime (2008-2016) we obtain

$$B_1 = 1.1554, B_2 = -0.1007, B_3 = 1.2099, B_4 = -0.0061$$
$$\lambda_1 = 1.0388, \lambda_2 = 0.1105$$

Now the model exhibits saddle-path stability, with one stable and one unstable real root. Under the proportional income taxation regime, which features a lower effective tax rate, hours are relocated away from home production, and towards the market sector. Therefore, aggregate output composition changes toward a higher share of market production, which is produced using a constant-returns-to-scale technology. We discuss the results for the (lack of) indeterminacy in detail in the following section.

6 Discussion

In this section we argue that the model discussed in this paper with a non-market sector is an isomorphic problem to a setup with increasing returns to scale and/or sector-specific externality. This is because total output in this framework is the sum of market output and home production. Market output is produced using a Cobb-Douglas function, which features constant returns to scale (CRS); On the other hand, production function in the non-market (”home”) sector features a decreasing returns to scale (DRS). However, when we aggregate over individual unregistered production, the home production function already features increasing returns to scale (IRS). The existence of IRS in this setup are easy to justify, as home production is always an available option for the household, and official GDP figures try to impute the size of non-market production in national accounts. Also, the non-market sector is treated differently than the official sector by statisticians, as taxes are based on registered production only. Thus the presence of a non-market sector generates externalities in production. Also, Farmer (1999) has shown that the presence of IRS can produce indeterminate equilibria, as long as the increasing returns are large enough. In this case the magnitude of the increasing returns is represented by the size of the non-market sector relative to overall production. Even though the two technologies produce the same goods, there is a different treatment in the model between the two sectors: The non-market output is non-tradable, and not directly observable. The other aspect of externality generated by the presence of home production in the model setup is that it is a non-competitive sector, and the implicit wage rate in the non-market sector differs from the marginal productivity of labor in that sector. This is because the sector is a monopolistic one: the firm faces a downward-sloping demand curve for labor in the non-market sector. Therefore, in equilibrium the wage rate in the home sector will feature a fixed mark-up $1/(1-\alpha) > 1$ over the marginal cost (or equivalently, the wage features a mark-up above the marginal productivity of labor). This pricing rule is obtained when we impose the zero-profit condition in the sector, which is in the spirit of free entry in models with monopolistic competition.

7 Conclusions

We show that in a exogenous growth model with non-market, or home-, production calibrated to Bulgarian data under the progressive taxation regime (1993-2007), the economy exhibits equilibrium indeterminacy due to the presence of home production. These results are in line with the findings in Benhabib and Farmer (1994, 1996) and Farmer (1999). Also, the findings in this paper are in contrast to Guo and Lansing (1988) who argue that progressive taxation works as an automatic stabilizer. Under the flat tax regime (2008-16), the economy calibrated to Bulgarian data displays saddle-path stability. The decrease in the average effective tax rate addresses the indeterminacy issue and eliminates the "stable focus" dynamics.

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Appendix: Equilibrium System

$$\frac{1}{c_{mt} + \theta A_{nt} h_{nt}^{1-\alpha}} = \lambda_t (1 + \tau^c) \tag{7.1}$$

$$\frac{\gamma}{1 - h_{mt} - h_{nt}} = \lambda_t [1 - (1 + \phi)\tau_{mt}] w_{mt} \tag{7.2}$$

$$\frac{\theta(1-\alpha) A_{nt} h_{nt}^{-\alpha}}{c_{mt} + \theta A_{nt} h_{nt}^{1-\alpha}} = \frac{\gamma}{1 - h_{mt} - h_{nt}} + \lambda_t w_{nt} \tag{7.3}$$

$$\lambda_t = \beta \lambda_{t+1} \left[1 + [1 - (1 + \phi)\tau_{m,t+1}] r_{t+1} - \delta \right] \tag{7.4}$$

$$w_{mt} = (1 - \alpha) \frac{y_{mt}}{h_{mt}} \tag{7.5}$$

$$r_t = \alpha \frac{y_{mt}}{k_t} \tag{7.6}$$

$$w_{nt} = A_{nt} h_{nt}^{-\alpha} \tag{7.7}$$

$$y_{mt} = A_{mt} k_t^{\alpha} h_{mt}^{1-\alpha} \tag{7.8}$$

$$y_{nt} = A_{nt} h_{nt}^{1-\alpha} \tag{7.9}$$

$$y_t = y_{mt} + y_{nt} \tag{7.10}$$

$$g_t^c = g^{cy} y_{mt} \tag{7.11}$$

$$\tau(y_{mt}) = \eta \left(\frac{y_{mt}}{y_m} \right)^{\phi} \tag{7.12}$$

$$\tau^c c_{mt} + \tau(y_{mt})[r_t k_t + w_{mt} h_{mt}] = g_t^c + g_t^t. \tag{7.13}$$