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Merger and Acquire of Series: A New Approach of Time Series Modeling

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Abstract

Present paper proposes an autoregressive time series model to study the behaviour of merger and acquire concept which is equally important as other available theories like structural break, de-trending etc. The main motivation behind newly proposed merged autoregressive (M-AR) model is to study the impact of merger in the parameters as well as acquired series. First, we recommend the estimation setup using popular classical least square and posterior distribution under Bayesian method with different loss function. Then, we obtain Bayes factor, full Bayesian significance test and credible interval to know the significance of the merger series. A simulation as well as empirical study is illustrated.

Keywords: Autoregressive model, Break point, Merger series, Bayesian inference

JEL Classification: C32, G34, C11

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1. Introduction

Time series models are preferred to analyze and establish the functional relationship considering it is an own dependence (Box and Jenkins (1970), Newbold (1983)) as well as some other covariate(s)/ explanatory series which alike parallel influence the series. However, these covariates may not survive for long run because of merged with dependent series. Such type of functional relationship is not explored by researchers yet but there are so many linear or non-linear models proposed in time series to analysis in a distinctly circumstances see Chan and Tong (1986), Engle (1982), Haggan and Ozaki (1981), Chon and Cohen (1997). On the basis of efficiency and accuracy, preferred time dependent model is chosen of further analysis and then do the forecasting. In daily real-life situations, we have a time series which is recorded as a continuous process for every business and organization. This plays very important role to analyze the economic development of the organization as well as nation. In present competitive market, all financial institutions feed upon the growth of their business by utilizing the available information and follow some basic business principles. Last few decades, rate of consolidations has been increasing tremendously to achieve the goal of higher profitability and widen business horizon. For this, higher capability institutions have a significant impact directly to weaker institutions. With the change on market strategies, some financial institutions are continuously working as well as growing well but there are few firms which are not efficiently operating as per public/state/owner's need and may be acquired by other strong company or possibly consolidated voluntarily or forcedly. For that reason, merger is a long run process to combine two or more than two companies freely which are having better understanding under certain condition. Sometimes strong company secures the small companies due to not getting high-quality performance in the market and also covers it's financial losses. Then, these companies are voluntarily merged in well-established company to meet out economical and financial condition with inferior risk.

In last few decades, researchers are taking inference to do research in the field of merger concept for the development of business and analyzed the impact and/or performance after the merger. Lubatkin (1983) addressed the issues of merger and shows benefits related to the acquiring firm. Healy *et al.* (1992) examined post-acquisition performance for the 50 largest U.S. mergers and showed significant improvements in asset productivity relative to their industries, leading to higher operating cash flow returns. This performance improvement is particularly strong for firms with highly overlapping businesses. Berger *et al.* (1999) provided a comprehensive review of studies for evaluating mergers and acquisitions (M&As) in banking industry. Maditinos *et al.* (2009) investigated the short as well as long merger effects of two banks and it's performance was recorded from the balance sheet.

Golbe and White (1988) discussed time dependent series of M&As and used OLS and 2SLS estimates to see the expected changes in future and concluded that merger series strongly follows autoregressive pattern. They also employed time series regression model to observe the simultaneous relationship between mergers and exogenous variables. Choi and Jeon (2011) applied time series econometric tools to investigate the dynamic impact of aggregate merger activity in US economy and found that macroeconomic variables and various alternative measures have a long-run equilibrium relationship at merger point. They also observed the most important macroeconomic variables which determine the US merger activity. Rao *et al.* (2016) studied the M&As in emerging markets by investigating post-M&A performance of ASEAN companies. They found that decrease in performance is particularly significant for M&As and have high cash reserves. Pandya (2017) measured the trend in mergers and acquisitions activity in manufacturing and non-manufacturing sector of India with the help of time series analysis and recorded the impact of merger by changes with government policies and political factors.

The above literatures have discussion on economical and financial point of view whereas merged series can be explored to know the dependence on time as well as own past observations. So, merger concept may be analyzed to model the series because merger of firms or companies are very specific due to failure of a firm or company. However, this is almost untouched yet for forecasting purpose. Time series model are most useful concept for forecasting. Both theoretical and empirical findings in existing literature argued that merger is effective for economy both positively and negatively as per limitations under reference (see Bates and Santerre (2000), Rao *et al.* (2016)). Therefore, a time series model is developed to model the merger process and show the appropriateness and effectiveness of the methodology in present manuscript. We have studied an autoregressive model to construct a new time model which accommodate the merger/acquire of series. First proposed the estimation methods in both classical and Bayesian framework then tested the effectiveness of the merger model using various significance tests. The performance of constructed model is demonstrated for recorded series of merger of mobile banking transaction series of SBI and its associate banks. A simulation study is also carried out to get more generalized idea for the model.

2. Merger Autoregressive (M-AR) Model

Let us consider $\{y_t$: $t = 1, 2, \ldots\ldots, T\}$ is a time series from ARX(p_1) model associated with k time dependent explanatory variables up to a certain time point called merger time T_m. After a considerable period, associated variables are merged in the dependent series as AR model with different order p_2. Then, the form of time series merger model is

$$y_t = \begin{cases} \theta_1 + \sum_{i=1}^{p_1} \phi_{1i} y_{t-i} + \sum_{m=1}^{k} \sum_{j=1}^{r_m} \delta_{mj} z_{m,t-j} + \varepsilon_t & t \le T_m \\ \theta_2 + \sum_{i=1}^{p_2} \phi_{2i} y_{t-i} + \varepsilon_t & t > T_m \end{cases} \tag{1}$$

where δ_m is merging coefficient of m^{th} series/variable and ε_t assumed to be *i.i.d.* normal random variable. Without loss of generality one may assume the number of merging series k as well as their merger time T_m and orders ($p_{i:}$ i=1, 2) to be known. Model (1) can be casted in matrix notation before and after the merger as follows

$$Y_{T_m} = \theta_1 l_{T_m} + \beta_1 X_{T_m} + \delta Z_{T_m} + \varepsilon_{T_m} \tag{2}$$

$$Y_{T-T_m} = \theta_2 l_{T-T_m} + \beta_2 X_{T-T_m} + \varepsilon_{T-T_m} \tag{3}$$

Combined eq^n (2) and eq^n (3) in vector form, produce the following equation

$$Y = l\theta + X\beta + Z\delta + \varepsilon \tag{4}$$

where

$$X_{T_m} = \begin{pmatrix} y_0 & y_{-1} & \cdots & y_{1-p_1} \\ y_1 & y_0 & \cdots & y_{2-p_1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{T_m-1} & y_{T_m-2} & \cdots & y_{T_m-p_1} \end{pmatrix} \quad X_{T-T_m} = \begin{pmatrix} y_{T_m} & y_{T_m-1} & \cdots & y_{T_m+1-p_2} \\ y_{T_m+1} & y_{T_m} & \cdots & y_{T_m+2-p_2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{T-1} & y_{T-2} & \cdots & y_{T-p_2} \end{pmatrix} \quad X = \begin{pmatrix} X_{T_m} & 0 \\ 0 & X_{T-T_m} \end{pmatrix}$$

$$Z_{T_m}^{m} = \begin{pmatrix} Z_{m,0} & Z_{m,-1} & \cdots & Z_{m,1-r_m} \\ Z_{m,1} & Z_{m,0} & \cdots & Z_{m,2-r_m} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{m,T_m-1} & Z_{m,T_m-2} & \cdots & Z_{m,T_m-r_m} \end{pmatrix} \quad Z_{T_m} = \begin{pmatrix} Z_{T_m}^1 & Z_{T_m}^2 & \cdots & Z_{T_m}^k \end{pmatrix} \quad Z = \begin{pmatrix} Z_{T_m} \\ 0 \end{pmatrix}$$

$$\beta_1 = \begin{pmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1p_1} \end{pmatrix}' \quad \beta_2 = \begin{pmatrix} \phi_{21} & \phi_{22} & \cdots & \phi_{2p_2} \end{pmatrix}' \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \quad \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

$$l = \begin{pmatrix} l_{T_m} & 0 \\ 0 & l_{T-T_m} \end{pmatrix} \quad \delta_m = \begin{pmatrix} \delta_{m1} & \delta_{m2} & \cdots & \delta_{mr_m} \end{pmatrix}' \quad \delta = \begin{pmatrix} \delta_1 & \delta_2 & \cdots & \delta_k \end{pmatrix}'$$

Model (4) is termed as merged autoregressive (M-AR(p_1, m, p_2)) model. The purpose behind M-AR model is to make an impress about merger series with acquisition series.

3. Inference for the Problem

The fundamental inference of any research is to utilize the given information in a way that can easily understand and describe problem under study. In time series, one may be interested to draw inference about the structure of model through estimation as well as conclude the model by testing of hypothesis. Thus, objective of present study is to establish the estimation and testing procedure for which model can handle certain particular situation.

3.1 Estimation under Classical Framework

Present section considers well known regression based method namely, classical least square estimator (OLS). For M-AR model, parameters of interest are *θ, β* and *δ*. To make the model more compact, one can write model (4) in further matrix form as

$$Y = \begin{pmatrix} l & X & Z \end{pmatrix} \begin{pmatrix} \theta \\ \beta \\ \delta \end{pmatrix} + \varepsilon = W\Theta + \varepsilon \tag{5}$$

For a given time series, estimating parameter(s) by least square and its corresponding sum of square residuals is given as

$$\hat{\Theta} = \begin{pmatrix} \hat{\theta} \\ \hat{\beta} \\ \hat{\delta} \end{pmatrix} = (W'W)^{-1} W'Y \tag{6}$$

and

$$SSR = \left(Y - W\hat{\Theta}\right)' \left(Y - W\hat{\Theta}\right) = \left(Y - W(W'W)^{-1}W'Y\right)' \left(Y - W(W'W)^{-1}W'Y\right)$$

3.2 Estimation under Bayesian Framework

Prior function provides available information about unknown parameters. Let us consider an informative conjugate prior distribution for all parameters of the model. For intercept, autoregressive and merger coefficient, adopt multivariate normal distribution having different mean but common variance depending upon the length of vector and error variance, assume inverted gamma prior $\sigma^2 \sim IG(a,b)$. Utilizing these priors, we may obtain the joint prior distribution

$$\Pi(\Theta) = \frac{b^a \left(\sigma^2\right)^{-\left(\frac{\sum_{m=1}^{k} r_m + p_1 + p_2}{2} + a + 2\right)}}{\left(2\pi\right)^{\frac{\sum_{m=1}^{k} r_m + p_1 + p_2 + 2}{2}} \Gamma(a)} \exp\left[-\frac{1}{2\sigma^2} \left\{ \begin{array}{l} (\theta-\mu)' I_2^{-1} (\theta-\mu) + (\beta-\gamma)' I_{p_1+p_2}^{-1} (\beta-\gamma) \\ + (\delta-\alpha)' I_R^{-1} (\delta-\alpha) + 2b \end{array} \right\} \right] \quad (7)$$

Under the given error assumption, likelihood function for observed series is

$$L(\Theta \mid y) = \frac{\left(\sigma^2\right)^{-\frac{T}{2}}}{\left(2\pi\right)^{\frac{T}{2}}} \exp\left[-\frac{1}{2\sigma^2} \left\{ (Y - l\theta - X\beta - Z\delta)' (Y - l\theta - X\beta - Z\delta) \right\} \right] \quad (8)$$

Under Bayesian approach, posterior distribution can be obtained from the joint prior distribution with combined information of observed series. For the proposed model, posterior distribution having the form

$$\Pi(\Theta \mid y) = \frac{b^a \left(\sigma^2\right)^{-\left(\frac{T+R+p_1+p_2}{2} + a + 2\right)}}{\left(2\pi\right)^{\frac{T+R+p_1+p_2}{2}+1} \Gamma(a)} \exp\left[-\frac{1}{2\sigma^2} \left\{ (Y - l\theta - X\beta - Z\delta)' (Y - l\theta - X\beta - Z\delta) + (\theta-\mu)' I_2^{-1} (\theta-\mu) + (\beta-\gamma)' I_{p_1+p_2}^{-1} (\beta-\gamma) + (\delta-\alpha)' I_R^{-1} (\delta-\alpha) + 2b \right\} \right] \quad (9)$$

Here, we are interested to estimate the parameters of the model under Bayesian framework and get the conditional posterior distribution:

$$\theta \big| \beta, \delta, \sigma^2, y \sim MVN\left(\left((Y - X\beta - Z\delta)' l + \mu' I_2^{-1} \right) \left(l'l + I_2^{-1} \right)^{-1}, \left(l'l + I_2^{-1} \right)^{-1} \sigma^2 \right) \quad (10)$$

$$\beta \big| \theta, \delta, \sigma^2, y \sim MVN\left(\left((Y - l\theta - Z\delta)' X + \gamma' I_{p_1+p_2}^{-1} \right) \left(X'X + I_{p_1+p_2}^{-1} \right)^{-1}, \left(X'X + I_{p_1+p_2}^{-1} \right)^{-1} \sigma^2 \right) \quad (11)$$

$$\delta \big| \theta, \beta, \sigma^2, y \sim MVN\left(\left((Y - l\theta - X\beta)' Z + \alpha' I_R^{-1} \right) \left(Z'Z + I_R^{-1} \right)^{-1}, \left(Z'Z + I_R^{-1} \right)^{-1} \sigma^2 \right) \quad (12)$$

$$\sigma^2 \big| \theta, \beta, \delta, y \sim IG\left(\frac{T + R + p_1 + p_2}{2} + a + 1, S \right) \quad (13)$$

where

$$S=\frac{1}{2}\Big[\left(Y-l\theta-X\beta-Z\delta\right)^{'}\left(Y-l\theta-X\beta-Z\delta\right)+\left(\theta-\mu\right)^{'}I_2^{-1}\left(\theta-\mu\right)$$
$$+\left(\beta-\gamma\right)^{'}I_{p_1+p_2}^{-1}\left(\beta-\gamma\right)+\left(\delta-\alpha\right)^{'}I_R^{-1}\left(\delta-\alpha\right)+2b\Big]$$

From a decision theory view point, for selection of optimal estimator, a loss function must be specified and is used to represent a penalty associated with each possible estimate. Since, there is no specific analytical procedure that allows us to identify the appropriate loss function. Usually, researchers reviewed various loss functions for better understanding. Therefore, we have considered following loss function (1) Squared Error Loss Function (SELF), (2) Linex Loss function (LLF), and (3) Absolute Loss Function (ALF) (Ali et al. (2013)), which are listed in table given below:

	SELF	LLF	ALF
Loss Function	$\left(\theta-\hat{\theta}\right)^2$	$\exp\left\{c\left(\hat{\theta}-\theta\right)\right\}-c\left(\hat{\theta}-\theta\right)-1$	$\left\lvert\theta-\hat{\theta}\right\rvert$
Bayes Estimator	$E\left(\theta\mid y\right)$	$-\frac{1}{c}\ln E\left(e^{-c\theta}\right)$	$Median\left[\Pi\left(\theta\mid x\right)\right]$

Considering above loss functions, we are not getting closed form expressions of Bayes estimators. Hence, Gibbs sampling, an iterative procedure is used to get the approximate values of the estimators using conditional posterior distribution. The credible interval is also computed using MCMC method proposed by Chen and Shao (1999).

3.3 Significance Test for Merger Coefficient

This section provides testing procedure to test the impact of merger series in model and targeting to analysis the impact on model as associate series may be influencing the model. The merger may have a positive or negative impact. Therefore, null hypothesis is assumed that merger coefficients are equal to zero H_0: $\delta=0$ against the alternative hypothesis that merger has a significant impact to the observed series H_1: $\delta\neq0$. Under the null and alternative hypothesis, models are as

Under $H_0 : Y = l\theta + X\beta + \varepsilon$ (14)

Under H_1: $Y = l\theta + X\beta + Z\delta + \varepsilon$ (15)

There are several Bayesian methods to handle the problem of testing the hypothesis. The commonly used testing strategy is Bayes factor, full Bayesian significance test and test based on credible interval. Here, one can easily understand the seriousness of appropriate significance test. Bayes factor is the ratio of posterior probability under null versus alternative hypothesis, notation given as:

$$BF_{10} = \frac{P(y \mid H_1)}{P(y \mid H_0)} \tag{16}$$

The Bayes Factor is obtained by the using of posterior probability under null hypothesis is

$$P(y \mid H_0) = \frac{b^a |A_1|^{-\frac{1}{2}} |A_2|^{-\frac{1}{2}} \Gamma\left(\frac{T}{2} + a\right)}{(2\pi)^{\frac{T}{2}} \Gamma(a) \left(\frac{S_0}{2}\right)^{\frac{T}{2}+a}} \tag{17}$$

and posterior probability under alternative hypothesis is

$$P(y \mid H_1) = \frac{b^a |A_1|^{-\frac{1}{2}} |A_2|^{-\frac{1}{2}} |A_3|^{-\frac{1}{2}} \Gamma\left(\frac{T}{2} + a\right)}{(2\pi)^{\frac{T}{2}} \Gamma(a) \left(\frac{S_1}{2}\right)^{\frac{T}{2}+a}} \tag{18}$$

where

$$A_1 = \left(l'l + I_2^{-1}\right)$$

$$A_2 = X'X + I_{p_1+p_2}^{-1} - X'lA_1^{-1}l'X$$

$$A_3 = Z'Z + I_R^{-1} - Z'lA_1^{-1}l'Z - \left(Z'X - Z'lA_1^{-1}l'X\right)' A_2^{-1}\left(Z'X - Z'lA_1^{-1}l'X\right)$$

$$B_{21}' = Y'X + \gamma' I_{p_1+p_2}^{-1} - \left(Y'l + \mu' I_2^{-1}\right)' A_1^{-1} l' X$$

$$B_3 = Y'Z + \alpha' I_R^{-1} - \left(Y'l + \mu' I_2^{-1}\right)' A_1^{-1} l' Z - B_{21}' A_2^{-1}\left(Z'X - Z'lA_1^{-1}l'X\right)$$

$$C_1 = \left(Y - X\beta\right)' l + \mu' I_2^{-1}$$

$$S_0 = Y'Y + \gamma' I_{p_1+p_2}^{-1}\gamma + \mu' I_2^{-1}\mu + 2b - \left(Y'l + \mu' I_2^{-1}\right)' A_1^{-1}\left(Y'l + \mu' I_2^{-1}\right) - B_{21}' A_2^{-1} B_{21}$$

$$S_1 = S_0 + \alpha' I_R^{-1}\alpha - B_3' A_3^{-1} B_3$$

Using the Bayes factor, one can easily taken decision regarding the acceptance or rejection of hypothesis. For small value of BF_{10}, leads to rejection of alternative hypothesis. With the help of BF_{01}, posterior probability of H_1 is obtained for the given data which is

$$PP = P\left(H_1 \mid y\right) = \left[1 + BF_{10}^{-1}\right]^{-1} \tag{19}$$

Sometimes, researchers may find the credible interval for a specified value in which rejection of null hypothesis depends upon the fact that how many estimated coefficients fall outside the interval. The credible intervals are highest posterior density which can be obtained from posterior density of the critical values and most of the time, posterior density expressions are not obtainable in closed form. Therefore, an alternative procedure is used to find out the credible region and so decision can be taken easily. Given $\alpha \in (0, 1)$, highest posterior density (HPD) region with a posterior probability α, is defined as

$$HPD = \left\{\delta \in R;\, P\left(\delta \mid y\right) \geq \alpha\right\} \quad s.t.\, P\left(HPD \mid y\right) = \alpha \tag{20}$$

Recently, a new Bayesian measure of evidence is used by researchers for choice of model or hypothesis testing named full Bayesian significance test (FBST). According to de Bragança Pereira and Stern (1999),who developed FBST test to measure the evidence in favour of a null hypothesis H_0 whenever it is large. For testing the presence of merger series in AR model, we also use FBST and evidence measure is defined as $Ev = 1-\gamma$ under the assumption that

$$\gamma = P\big(\delta : \pi(\delta \mid y) > \pi(\delta_0 \mid y)\big) \tag{21}$$

4. Simulation Study

To demonstrate the merger concept in proposed time series model, a simulation study is illustrated. In simulation, a series is generated based on initial information about unknown quantity. We start our analysis using the generated series form the *M-AR* model for different sizes of the series $T = \{100, 200, 300\}$ with different merger time T_M. For each generated series, initial value of parameters is assumed for the model which is defined as

$$y_t = \begin{cases} 0.2 + 0.5y_{t-1} + 0.05z_{1,t-1} + 0.1z_{2,t-1} + 0.15z_{3,t-1} + \varepsilon_t & t \le T_m \\ 0.3 + 0.3y_{t-1} + 0.5y_{t-2} + \varepsilon_t & t > T_m \end{cases} \tag{22}$$

with error term is $N(0, 2)$. For simplicity, merger series also follows AR(1) process with intercept term is 0.05. The initial value of $y_0 = 5$ and $z = \{1.9, 2.7, 1.5\}$ are assumed to initiate the process. For recording the results of the expressions of posterior density of each model parameter, an analytical and numerical technique is applied. As the expression is not in closed form so Gibbs sampling algorithm with 10,000 replications has been used to approximate the value of conditional posterior density for parameter estimation and posterior probability to test the hypothesis associated therein. To get more generalized idea of M-AR model, compared different methods of estimation under classical and Bayesian approach and reported in terms of mean square error (MSE) and absolute bias (AB)by Figures A1-A9 in the Appendix.

From the figures, it is recorded that as size of the series increases MSE and AB are decreasing for different time point of merger. It is also observed that OLS estimator performance is poor as compared to Bayesian estimator. But when we make comparison between the loss function, then both symmetric SELF and ALF shows better results in comparison to both OLS as well as asymmetric loss function except error variances. Bayes estimator under SELF is equally applicable as ALF in estimating the parameters since both the estimators show similar magnitudes for their MSE. Hence, choice of loss function is concerned upon the nature of parameters and as some times its how same results approximately. From the figure, it is also recorded that with the increase on size of the merger series, MSE and AB decreases before the merger time whereas increases MSE and AB of estimator after the merger times. Further, we also computed confidence interval based on different sample series and different values of merger points. A highest posterior interval is calculated based on 10,000 replications to obtain the upper and lower bound of the parameter at 5% level of significance which are reported in Tables 1-5.

Table 1: Confidence interval for intercept term with varying merger series

		θ_1				θ_2			
T	TM	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS
100	20	(-0.47-0.93)	(-0.58-0.80)	(-0.48-0.93)	(-0.51-1.37)	(-0.06-0.76)	(-0.07-0.71)	(-0.06-0.75)	(-0.07-0.93)
	40	(-0.24-0.70)	(-0.25-0.67)	(-0.23-0.70)	(-0.25-0.83)	(-0.12-0.93)	(-0.14-0.86)	(-0.12-0.94)	(-0.12-1.19)
	60	(-0.15-0.61)	(-0.17-0.59)	(-0.15-0.62)	(-0.16-0.68)	(-0.24-1.10)	(-0.25-1.04)	(-0.24-1.17)	(-0.19-1.53)
	80	(-0.10-0.57)	(-0.12-0.55)	(-0.13-0.54)	(-0.14-0.57)	(-0.89-1.69)	(-1.12-1.55)	(-1.02-1.86)	(-0.80-2.27)
200	40	(-0.27-0.67)	(-0.29-0.63)	(-0.27-0.66)	(-0.28-0.79)	(0.07-0.63)	(0.07-0.61)	(0.08-0.63)	(0.07-0.70)
	80	(-0.11-0.53)	(-0.12-0.51)	(-0.11-0.53)	(-0.12-0.56)	(0.04-0.68)	(0.03-0.66)	(0.04-0.67)	(0.01-0.75)
	120	(-0.04-0.50)	(-0.05-0.49)	(-0.04-0.50)	(-0.05-0.51)	(-0.05-0.82)	(-0.03-0.80)	(-0.05-0.81)	(-0.08-0.97)
	160	(-0.01-0.48)	(-0.02-0.48)	(-0.01-0.48)	(-0.02-0.49)	(-0.31-1.05)	(-0.34-0.96)	(-0.31-1.05)	(-0.29-1.43)
300	60	(-0.14-0.64)	(-0.16-0.61)	(-0.15-0.63)	(-0.15-0.71)	(0.11-0.58)	(0.10-0.57)	(0.11-0.58)	(0.11-0.60)
	120	(-0.01-0.51)	(-0.02-0.50)	(-0.03-0.50)	(-0.03-0.51)	(-0.03-0.80)	(-0.05-0.75)	(-0.03-0.80)	(-0.05-0.98)
	180	(0.01-0.44)	(0.00-0.43)	(0.01-0.44)	(0.01-0.44)	(0.00-0.69)	(-0.01-0.66)	(0.00-0.69)	(0.03-0.87)
	240	(0.04-0.41)	(0.03-0.41)	(0.04-0.42)	(0.04-0.42)	(-0.11-0.94)	(-0.13-0.88)	(-0.20-0.87)	(-0.15-1.18)

Table 2: Confidence interval for AR(1) coefficients with varying merger series

		ϕ_{11}				ϕ_{21}			
T	TM	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS
100	20	(0.13-0.77)	(0.11-0.76)	(0.14-0.78)	(0.06-0.79)	(0.09-0.48)	(0.09-0.48)	(0.10-0.49)	(0.08-0.48)
	40	(0.22-0.71)	(0.21-0.70)	(0.22-0.71)	(0.20-0.71)	(0.03-0.51)	(0.02-0.50)	(0.03-0.51)	(0.02-0.51)
	60	(0.27-0.67)	(0.27-0.66)	(0.27-0.67)	(0.26-0.67)	(-0.03-0.56)	(-0.04-0.55)	(-0.02-0.57)	(-0.08-0.54)
	80	(0.30-0.64)	(0.29-0.64)	(0.29-0.64)	(0.29-0.64)	(-0.29-0.65)	(-0.33-0.61)	(-0.29-0.65)	(-0.32-0.66)
200	40	(0.25-0.71)	(0.24-0.71)	(0.25-0.71)	(0.24-0.72)	(0.16-0.43)	(0.16-0.43)	(0.16-0.43)	(0.16-0.43)
	80	(0.31-0.67)	(0.30-0.66)	(0.31-0.67)	(0.30-0.66)	(0.13-0.46)	(0.13-0.46)	(0.13-0.46)	(0.13-0.46)
	120	(0.33-0.63)	(0.33-0.63)	(0.33-0.63)	(0.33-0.63)	(0.07-0.47)	(0.07-0.48)	(0.07-0.48)	(0.07-0.48)
	160	(0.37-0.63)	(0.36-0.63)	(0.37-0.63)	(0.36-0.63)	(-0.05-0.56)	(-0.07-0.54)	(-0.04-0.57)	(-0.09-0.56)
300	60	(0.28-0.67)	(0.27-0.67)	(0.28-0.67)	(0.27-0.67)	(0.18-0.40)	(0.18-0.40)	(0.18-0.40)	(0.18-0.40)
	120	(0.35-0.64)	(0.35-0.64)	(0.35-0.65)	(0.35-0.65)	(0.06-0.47)	(0.05-0.46)	(0.06-0.47)	(0.05-0.48)
	180	(0.36-0.61)	(0.35-0.60)	(0.36-0.60)	(0.36-0.61)	(0.13-0.44)	(0.13-0.44)	(0.13-0.44)	(0.12-0.44)
	240	(0.39-0.61)	(0.39-0.61)	(0.39-0.61)	(0.39-0.61)	(0.04-0.50)	(0.03-0.49)	(0.03-0.50)	(0.03-0.51)

Table 3: Confidence interval for AR(2) coefficient and error variance with varying merger series

		ϕ_{22}				σ^2			
T	TM	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS
100	20	(0.26-0.63)	(0.26-0.63)	(0.26-0.63)	(0.26-0.64)	(1.20-2.05)	(1.19-2.01)	(1.19-2.03)	(1.50-2.61)
	40	(0.22-0.67)	(0.22-0.66)	(0.22-0.67)	(0.20-0.66)	(1.17-2.04)	(1.16-2.00)	(1.14-2.00)	(1.50-2.62)
	60	(0.12-0.67)	(0.11-0.66)	(0.12-0.67)	(0.10-0.66)	(1.17-2.07)	(1.15-2.03)	(1.16-2.05)	(1.43-2.60)
	80	(-0.06-0.72)	(-0.10-0.70)	(-0.08-0.71)	(-0.13-0.70)	(1.17-2.08)	(1.16-2.05)	(1.16-2.06)	(1.44-2.61)
200	40	(0.34-0.61)	(0.34-0.61)	(0.34-0.61)	(0.34-0.61)	(1.45-2.12)	(1.44-2.10)	(1.44-2.11)	(1.62-2.40)
	80	(0.31-0.61)	(0.30-0.61)	(0.31-0.62)	(0.30-0.61)	(1.43-2.11)	(1.45-2.12)	(1.45-2.12)	(1.62-2.39)
	120	(0.25-0.63)	(0.24-0.63)	(0.25-0.63)	(0.25-0.64)	(1.45-2.16)	(1.43-2.12)	(1.44-2.14)	(1.63-2.44)
	160	(0.12-0.67)	(0.11-0.67)	(0.12-0.67)	(0.09-0.66)	(1.42-2.17)	(1.41-2.15)	(1.40-2.15)	(1.64-2.49)
300	60	(0.37-0.60)	(0.37-0.60)	(0.37-0.60)	(0.36-0.59)	(1.58-2.17)	(1.57-2.15)	(1.56-2.15)	(1.71-2.36)
	120	(0.24-0.63)	(0.24-0.63)	(0.25-0.63)	(0.24-0.64)	(1.45-2.15)	(1.45-2.14)	(1.45-2.15)	(1.63-2.43)
	180	(0.32-0.62)	(0.32-0.62)	(0.32-0.62)	(0.32-0.62)	(1.58-2.20)	(1.57-2.18)	(1.53-2.14)	(1.70-2.38)
	240	(0.22-0.67)	(0.20-0.67)	(0.21-0.67)	(0.19-0.66)	(1.56-2.16)	(1.56-2.15)	(1.56-2.15)	(1.68-2.33)

Table 4: Confidence interval for merger coefficients with varying merger series

		δ_1				δ_2			
T	TM	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS
100	20	(-0.75-0.68)	(-0.82-0.63)	(-0.75-0.68)	(-0.83-0.79)	(-0.64-0.69)	(-0.71-0.64)	(-0.65-0.69)	(-0.78-0.74)
	40	(-0.50-0.42)	(-0.52-0.40)	(-0.50-0.42)	(-0.50-0.46)	(-0.44-0.48)	(-0.47-0.45)	(-0.45-0.47)	(-0.48-0.48)
	60	(-0.37-0.38)	(-0.39-0.37)	(-0.37-0.38)	(-0.39-0.38)	(-0.37-0.38)	(-0.39-0.36)	(-0.36-0.39)	(-0.38-0.39)
	80	(-0.30-0.33)	(-0.32-0.31)	(-0.31-0.33)	(-0.31-0.33)	(-0.31-0.32)	(-0.31-0.33)	(-0.31-0.32)	(-0.32-0.33)
200	40	(-0.49-0.47)	(-0.53-0.44)	(-0.49-0.47)	(-0.51-0.50)	(-0.45-0.45)	(-0.47-0.42)	(-0.44-0.45)	(-0.47-0.47)
	80	(-0.32-0.32)	(-0.36-0.29)	(-0.34-0.31)	(-0.33-0.33)	(-0.33-0.30)	(-0.34-0.29)	(-0.33-0.30)	(-0.32-0.32)
	120	(-0.27-0.25)	(-0.28-0.24)	(-0.27-0.25)	(-0.27-0.25)	(-0.26-0.28)	(-0.26-0.27)	(-0.26-0.28)	(-0.26-0.28)
	160	(-0.23-0.21)	(-0.24-0.21)	(-0.23-0.21)	(-0.24-0.21)	(-0.21-0.23)	(-0.22-0.23)	(-0.21-0.23)	(-0.21-0.23)
300	60	(-0.36-0.34)	(-0.38-0.33)	(-0.36-0.35)	(-0.37-0.35)	(-0.38-0.36)	(-0.40-0.34)	(-0.35-0.39)	(-0.37-0.39)
	120	(-0.23-0.27)	(-0.26-0.25)	(-0.23-0.27)	(-0.25-0.27)	(-0.23-0.28)	(-0.23-0.27)	(-0.22-0.28)	(-0.23-0.28)
	180	(-0.22-0.19)	(-0.23-0.19)	(-0.21-0.20)	(-0.21-0.20)	(-0.20-0.23)	(-0.21-0.22)	(-0.20-0.23)	(-0.21-0.22)
	240	(-0.19-0.18)	(-0.19-0.18)	(-0.18-0.18)	(-0.18-0.19)	(-0.17-0.18)	(-0.18-0.18)	(-0.17-0.18)	(-0.17-0.18)

Table 5: Confidence interval with varying merger series for δ_3

T	TM	CI_{SELF}	CI_{LLF}	CI_{ALF}	OLS
100	20	(-0.64-0.69)	(-0.71-0.64)	(-0.65-0.69)	(-0.78-0.74)
	40	(-0.44-0.48)	(-0.47-0.45)	(-0.45-0.47)	(-0.48-0.48)
	60	(-0.37-0.38)	(-0.39-0.36)	(-0.36-0.39)	(-0.38-0.39)
	80	(-0.31-0.32)	(-0.31-0.33)	(-0.31-0.32)	(-0.32-0.33)
200	40	(-0.45-0.45)	(-0.47-0.42)	(-0.44-0.45)	(-0.47-0.47)
	80	(-0.33-0.30)	(-0.34-0.29)	(-0.33-0.30)	(-0.32-0.32)
	120	(-0.26-0.28)	(-0.26-0.27)	(-0.26-0.28)	(-0.26-0.28)
	160	(-0.21-0.23)	(-0.22-0.23)	(-0.21-0.23)	(-0.21-0.23)
300	60	(-0.38-0.36)	(-0.40-0.34)	(-0.35-0.39)	(-0.37-0.39)
	120	(-0.23-0.28)	(-0.23-0.27)	(-0.22-0.28)	(-0.23-0.28)
	180	(-0.20-0.23)	(-0.21-0.22)	(-0.20-0.23)	(-0.21-0.22)
	240	(-0.17-0.18)	(-0.18-0.18)	(-0.17-0.18)	(-0.17-0.18)

From Tables 1-5, one can observe that minimum average width is achieved from LLF estimator as compared to others estimator. To compute the Bayes factor, we assumed that each prior probability is equally likely associated with null and alternative hypothesis. A 5% level is defined to calculate the FBST and credible interval test i.e. coverage probability (CP).

Table 6: Evidence measures for testing null hypotheses with varying T and T_M

T	TM	BF	PP	CP	FBST
100	20	2.27E+19	0.9459	0.9208	0.0004
	40	1.67E+43	0.9295	0.9093	0.0042
	60	8.35E+187	0.8459	0.8897	0.0212
	80	1.58E+299	0.5824	0.8900	0.1222
200	40	1.11E+03	0.9597	0.9292	0.0000
	80	7.65E+34	0.9605	0.9228	0.0004
	120	7.01E+150	0.8611	0.9200	0.0128
	160	5.67E+176	0.5977	0.9132	0.0818
300	60	4.97E+01	0.9668	0.9348	0.0000
	120	2.52E+04	0.9581	0.9303	0.0000
	180	1.09E+26	0.8791	0.9270	0.0004
	240	2.58E+134	0.6876	0.9222	0.0180

From Table 6, it is notice that if merger is occurred in the first quartile, impact is not much effect, but it is significant to reject the null hypothesis whereas in third quartile, strong correlation is examining in merger and acquire series using Bayes factor. The coverage probability is high with increase of size of series, but it is inversely proportional to size of merger series which can be seen in the results. Similarly, using FBST evidence measure, there is strong reject of null hypothesis for small value of merger points but as merger point occurs near the size of the series (T), substantial evidence is recorded against the null hypothesis.

5. Merger in Banking Industry: An application

It is well defined that banking sector has strong contribution in any economy. It has been adopted various approaches to smooth working in the global front. Merger and acquisition is one of the finest approaches of consolidation that offers potential growth in Indian banking. State bank of India (SBI) is the largest bank in India. Recently SBI merged with five of its associate banks and Bharatiya mahila bank is becoming the largest lender in the list of top 50 banks in the world. The combined base of SBI is expected to increase productivity, reduce geographical risk and enhance operating efficiency. In India, there are various channels to transfer the payment online. Mobile banking is one of the important channels to transfer the money using a mobile device which is introduced since 2002 and become popular after demonization as it is a very fast and effective performed using smart phone and tablet. For analysis of proposed model, we have taken monthly data series of mobile banking of SBI and its associate banks over the period from November 2009 to November 2017. Data series gives information about the total number of transactions with its total payment in a specific month for a fix bank. For analysis purpose, we have converted data into payment per transactions for the merger banks.

The objective of the proposed study is to observe the impact of merger series. First, fitted an autoregressive model to mobile banking series to find out the most prefer order (lag) of SBI and its associate merger banks and then study the inference. Table 7 shows the descriptive statistics and lag of AR model with estimated coefficients for each series. Once getting the lag (order) of each associate series, apply M-AR model to estimate the model parameters using OLS and Bayesian approach which are recorded in Table 8 and observed that there may be change in estimated value when considering merger in the series. From Tables 7-8, we observed that there is a negative change happened due to SBBJ and SBP series because the sign of coefficient value is transform whereas other remaining series have a positive impact but not much affects the SBI series. To know the impact of associate banks series, testing the presence of merged series and reported in Table 9. Table 9 explained the connection between associate banks with SBI and observed that banks merger has a significantly impact of SBI series and after the merger point, there is a decrease in the mobile banking transactions. All assumed test is correctly identifying the effect of merger.

Table 7: Descriptive statistics and order of the mobile banking series

Series	Mean	St. deviation	Skewness	Kurtosis	Order	ϕ_1	ϕ_2	ϕ_3
SBI	4.4983	8.3656	2.1974	3.5764	1	0.9297	-	-
SBBJ	0.7745	0.6569	2.4332	5.3273	2	1.0845	-0.2113	-
SBH	0.7125	0.8462	2.7081	6.6017	2	1.044	-0.1683	-
SBM	0.9295	0.8768	2.1176	4.0361	1	0.8934	-	-
SBP	0.985	1.1079	2.215	3.7352	3	0.7663	0.2626	-0.1646
SBT	0.8781	0.7335	2.432	5.5085	1	0.8909	-	-
M-SBI	10.2032	4.6229	0.4149	-1.8709	1	0.5768	-	-

Table 8: Bayes and OLS estimates based on mobile banking series

	SELF			LLF			ALF			OLS		
	θ_1	θ_2	σ^2	θ_1	θ_2	σ^2	θ_1	θ_2	σ^2	θ_1	θ_2	σ^2
	-0.1170	5.2540	2.9672	-0.1326	5.1345	2.8947	-0.1120	5.1070	3.1220	-0.2840	4.6410	2.3110
Series	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3
SBI	0.9630	-	-	0.9630	-	-	0.9590	-	-	0.9590	-	-
SBBJ	-2.7872	2.0816	-	-3.7244	1.3773	-	-2.0700	1.6700	-	-5.0600	4.6140	-
SBH	0.9102	-2.2504	-	0.4315	-2.7448	-	1.4500	-2.0230	-	1.7640	-3.9340	-
SBM	1.0344	-	-	0.9453	-	-	1.0180	-	-	1.4570	-	-
SBP	-0.7404	-0.1806	1.8968	-0.7770	-0.2423	1.8704	-0.8750	-0.2230	1.8920	-0.9260	0.0030	2.0160
SBT	0.3318	-	-	0.2763	-	-	0.1770	-	-	0.3870	-	-
M-SBI	0.3344	-	-	0.3341	-	-	0.3390	-	-	0.3180	-	-

Table 9: Testing the hypothesis for on mobile banking series and its confidence interval

	BF	PP	FBST
	1.53E+76	0.7467	0.0404
Series	ϕ_1	ϕ_2	ϕ_3
SBI	(0.96-1.66)	-	-
SBBJ	(-3.28-0.56)	(-5.06--5.06)	-
SBH	(0.60-6.86)	(-4.37-1.76)	-
SBM	(-5.21--3.93)	-	-
SBP	(-1.82-1.46)	(-0.55-0.49)	(1.29-2.26)
SBT	(-2.66-0.46)	-	-
M-SBI	(0.28-0.56)	-	-

6. Conclusion

Time series modeling, sole is to establish/know the dependency with past observation(s) as well as other associated observed series(s) which are partially or fully influencing the current observation. After merger, few series are not recorded due to discontinuation of series because of many reasons like inadequate performance, new technology changes, increasing market operation etc. This is dealt by various econometrician and policy makers and termed merger. Since few decades it’s becoming very popular to handle the problem of weaker organization to improve its functioning or acquire it to help the employees as well as continue the ongoing business. Therefore, a model is proposed in time series to classify the merger and acquire scenario in modeling. A classical and Bayesian inference is obtained for estimation and its confidence interval. Various testing methods are also used to observe the presence of merger series in the acquire series. Simulation study is verifying the use and purpose of model. Recently, SBI associate banks are merged in SBI to strengthen the Indian Banking. Thus, mobile banking data of these banks was used to analysis the empirical presentation of the model and recorded that merger has a significant effect for the SBI series in terms of reducing the transactions.

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Appendix

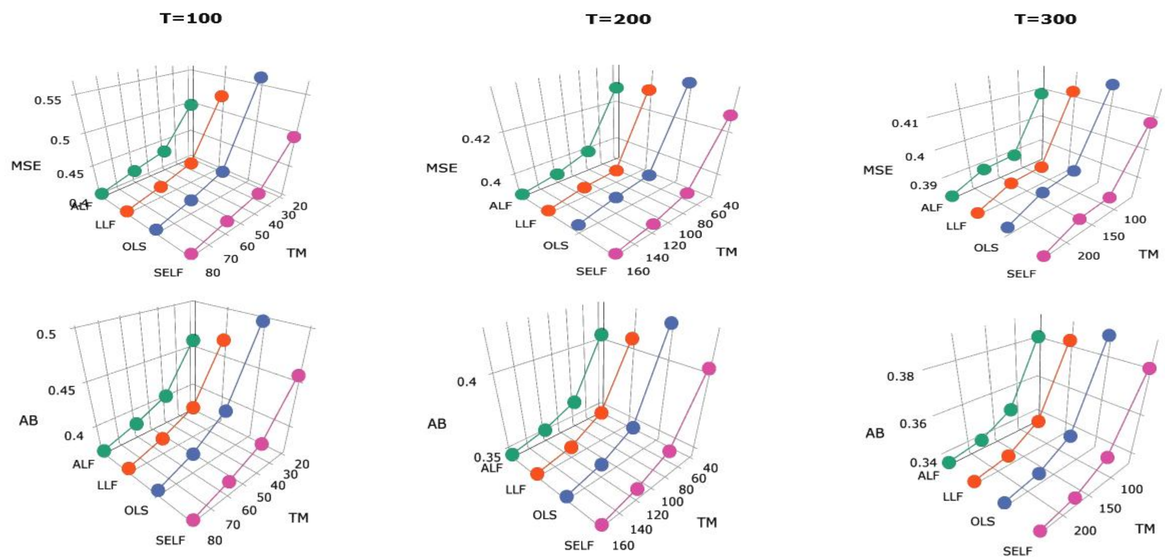

Figure A1: AB and MSE of the estimator θ_1, with varying T and T_M

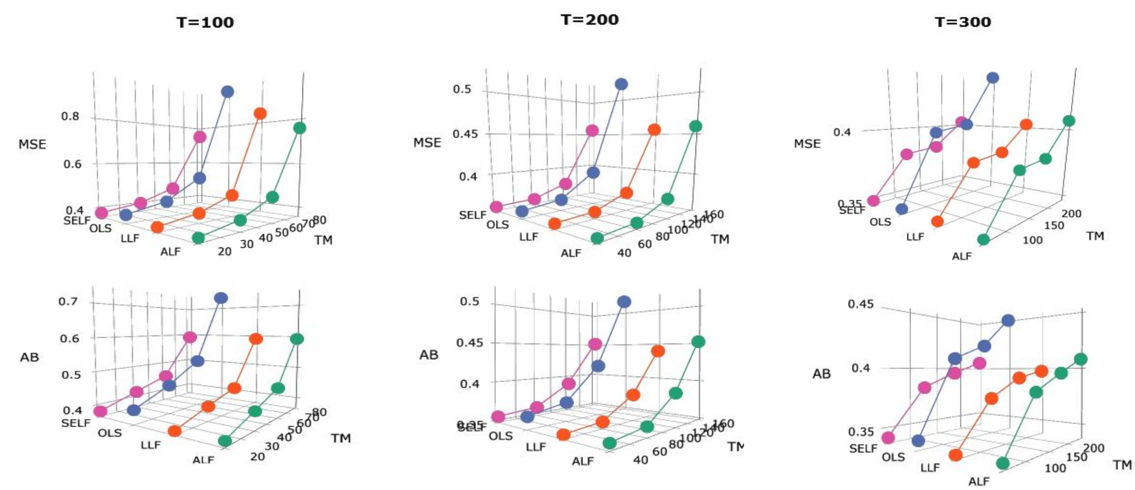

Figure A2: AB and MSE of the estimator θ_2, with varying T and T_M

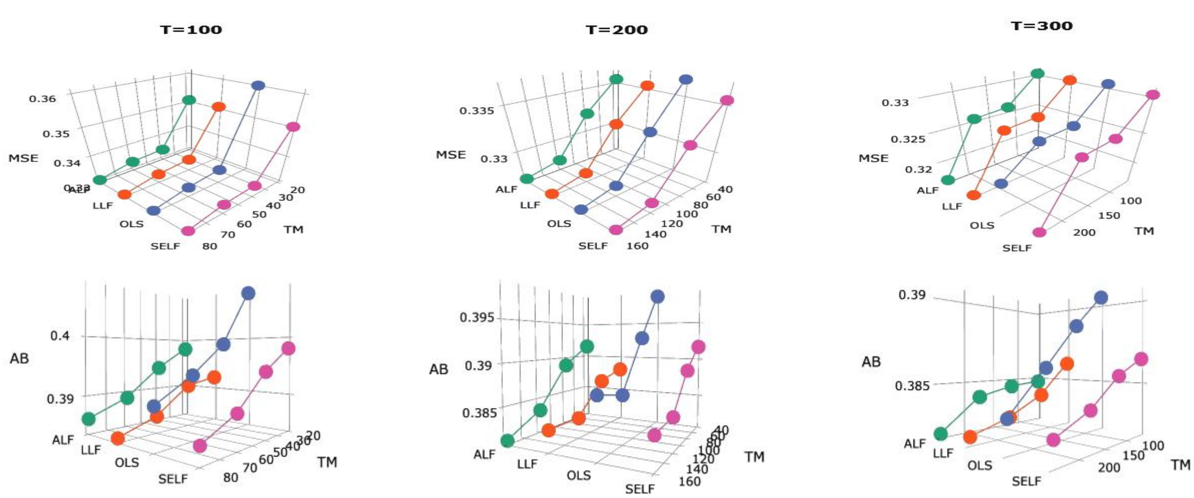

Figure A3: AB and MSE of the estimator ϕ_{11}, with varying T and T_M

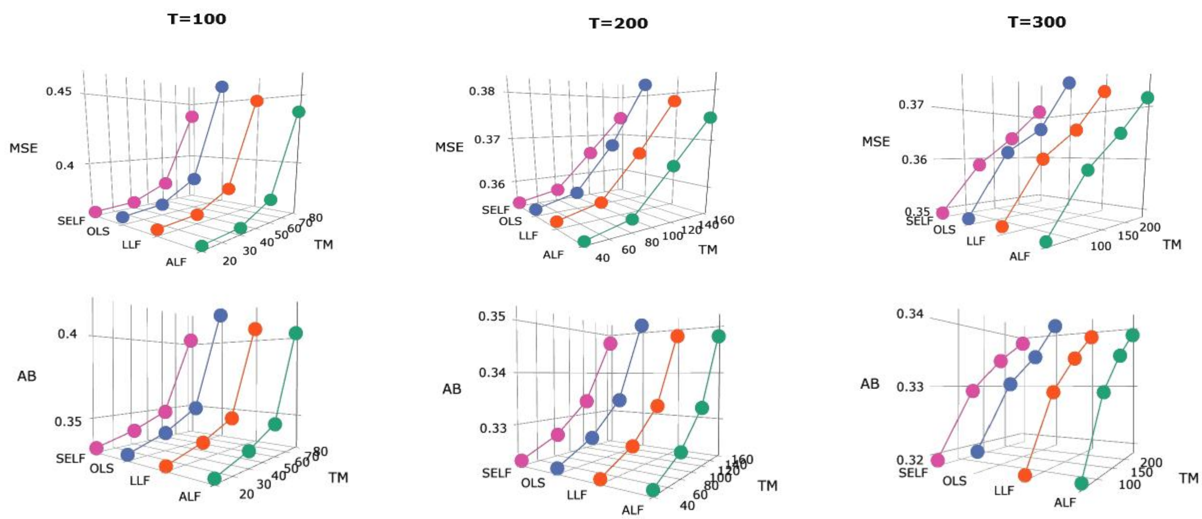

Figure A4: AB and MSE of the estimator ϕ_{21}, with varying T and T_M

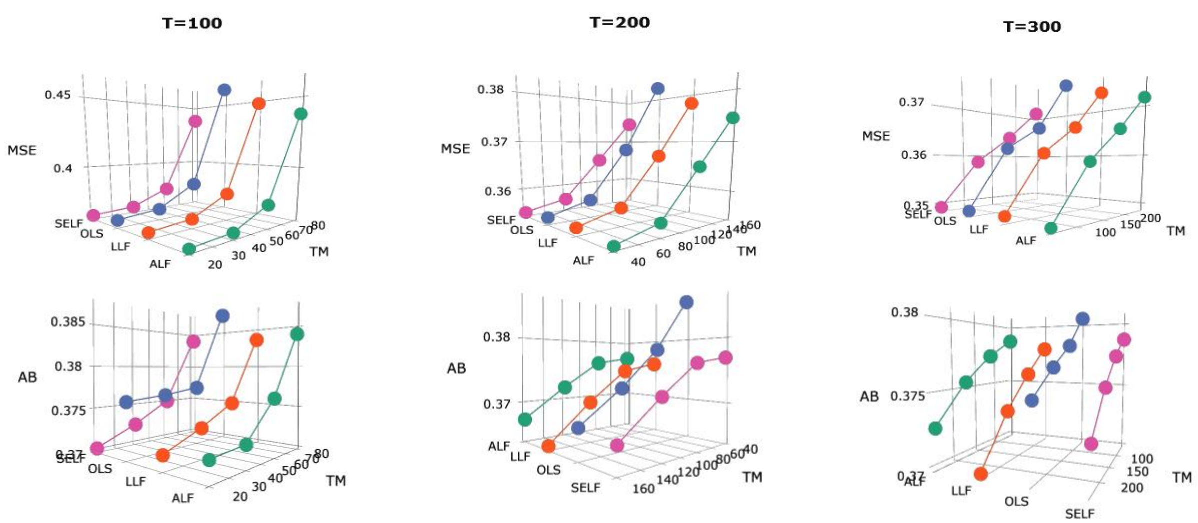

Figure A5: AB and MSE of the estimator ϕ_{22}, with varying T and T_M

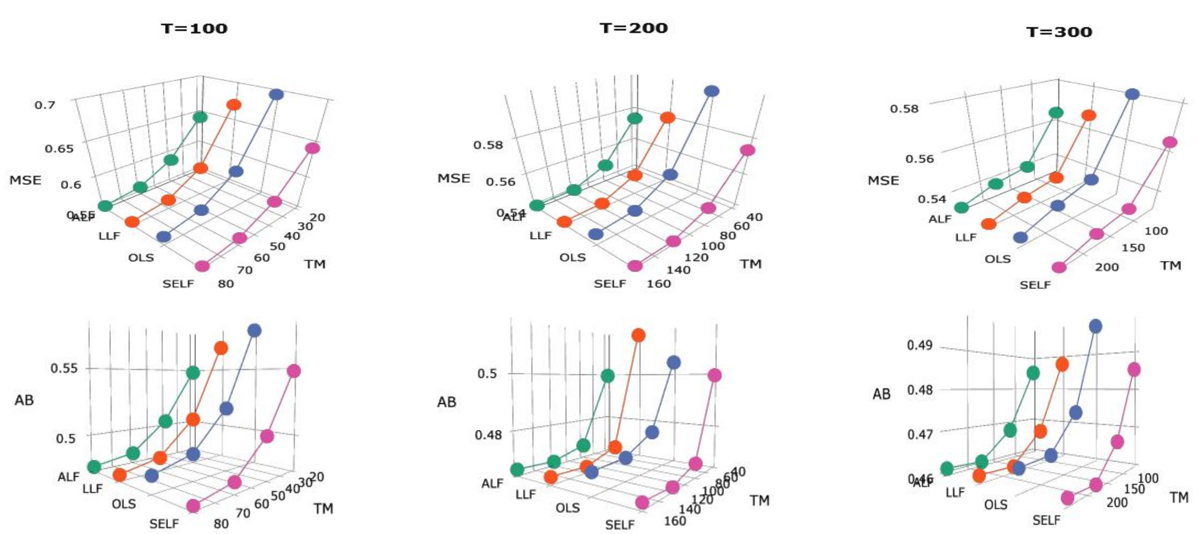

Figure A6: AB and MSE of the estimator δ_{11}, with varying T and T_M

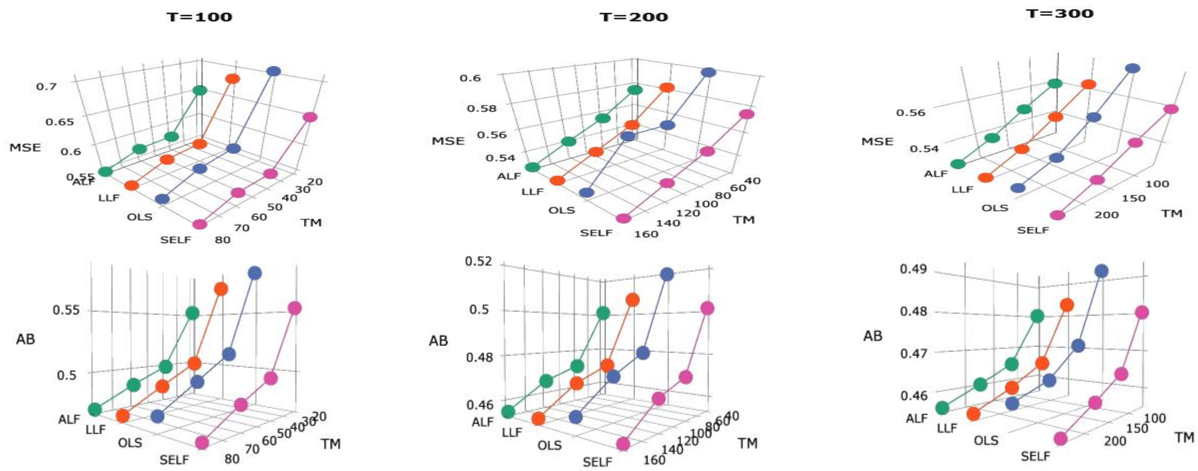

Figure A7: AB and MSE of the estimator δ_{21}, with varying T and T_M

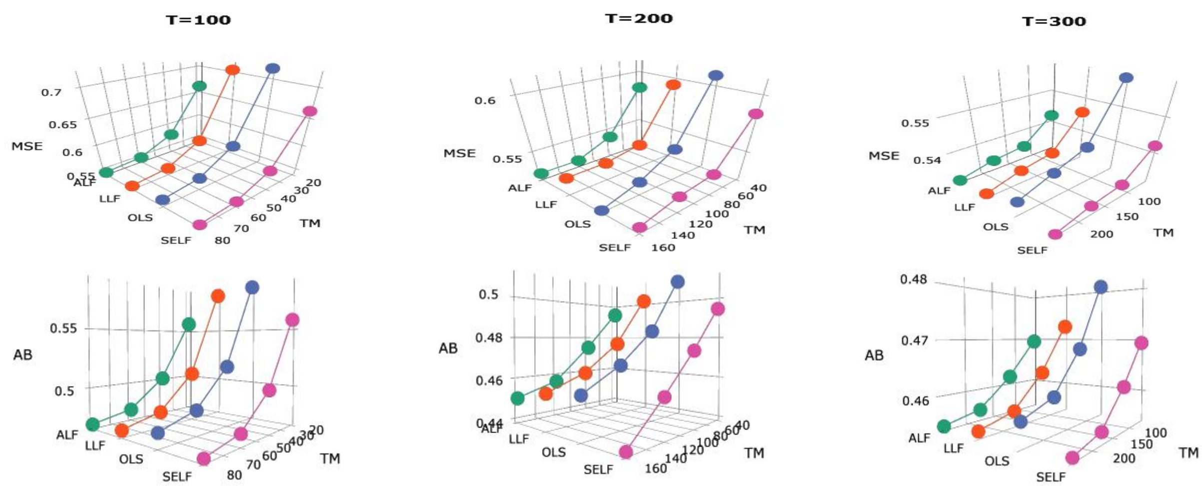

Figure A8: AB and MSE of the estimator δ_{31}, with varying T and T_M

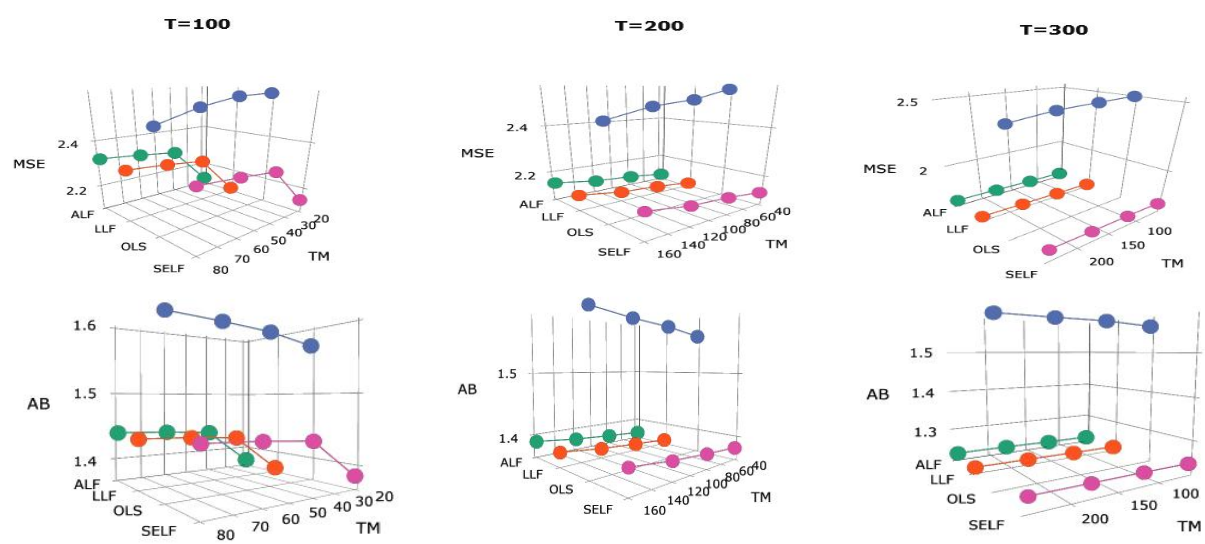

Figure A9: AB and MSE of the estimator σ^2, with varying T and T_M