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The Econometrics of Global Warming

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The Econometrics of Global Warming 1959-2020

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Abstract

Evidence-based policy re global warming is best relying on a relevant sample of data. Showing close correlation between CO2 and temperature over hundreds of thousands of years is irrelevant today. We choose a sample of annual data from 1959 to-date to provide some statistically robust stylized facts about the relationships between actual CO2 and temperature. Visually, there is a clear upward trend in both data. Time series analyses suggest that CO2 is difference-stationary and temperature is trend-stationary. Thus, the moments (mean, variance, etc.) of the data in *levels* are functions of time, which means that the correlation between the two variables may be spurious. Most importantly is that the variance of CO2 (and all greenhouse gases) are significantly smaller than the variance of temperature, hence they cannot explain the variations in temperature. We find no statistically robust evidence of correlation, long run co-variation, long run common trend, or common cycles between CO2 and temperature over a period of 60 years. Nonetheless, at most 40 percent of the variance of the Northern Hemisphere temperature is due to , 20 percent of the Southern Hemisphere, and much less of global temperature.

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1. Introduction

There has been growing scientific evidence that the *actual* increase in greenhouse gasses (GHG; CO_2, CH_4, N_2O, fluorinated gases) and global warming, i.e. temperature, are closely correlated over time, hence, the policy urgency to act faster to reduce greenhouse gases. Cook *et al.* (2013) says that 97.1 percent of articles' abstracts on global warming "endorsed the consensus position that humans are causing global warming." Lindsey and Dahlman (2020) wrote, "Though warming has not been uniform across the planet, the upward trend in the globally averaged temperature shows that more areas are warming than cooling. According to the National Oceanic and Atmospheric Administration (NOAA) 2019 Global Climate Summary, the combined land and ocean temperature has increased at an average rate of 0.07°C (0.13°F) per decade since 1880; however, the average rate of increase since 1981 (0.18°C / 0.32°F) is more than twice as great." There is no doubt that both temperature and CO2 level have been increasing.

In addition to correlation, the common consensus among scientists across many different disciplines is that (manmade) greenhouse gases *cause* an increase in global warming (increase in land and ocean temperature), hence the growing global popular campaign re climate change. Scientists use many different methods to measure temperature and greenhouse gases, e.g., ice coring, tree rings, balloons...etc. This literature is extraordinarily voluminous, runs across many disciplines, and readily available on the Internet, which we will not attempt to cite but we will restrict our citation to immediately relevant articles.

Scientists have used many different sources of data too, and many different techniques to show correlation between CO2 and temperature. A typical argument goes like this, "One of the most remarkable aspects of the paleoclimate record is the strong correspondence between temperature and the concentration of CO2 in the atmosphere observed during the glacial cycles of the past several hundred thousand years," and the graphical presentation in (Jouzel *et al.* 2007 and Lüthi *et al.* 2008). Figure (1) shows the temperature (light color) and (dark color) measured by the European Project for Ice Coring in Antarctica (EPICA). The graphical correlation is extraordinary. The relationship is so tight one needs do nothing else but believe it.

However, there are a number of issues in figure (1). First, given the significant correlation between CO2 and temperature for 800,000 years, obviously not all greenhouse gasses have been *manmade*. Second, the correlation between CO2 and temperature at zero years before present seems the same or lower than it was 100,000 years before present! Third, history seems to suggest that there are prolonged periods of low CO2 and low temperature levels, so what caused CO2 level to fall without any interventionist policy? Fourth, the graph is not a proof of causation. Could temperature cause CO2? Could causality be bi-directional, running in both directions? Fifth, however, most importantly for this paper, which is not about the science of climate warming *per se* but about using statistics to analyze scientific data of global warming, is that both CO2 and temperature are measured in *levels and have trends.* Trend in the data could render the correlation between the levels spurious. This paper focuses on the time series analysis of trend, and on how to calculate and estimate meaningful associations between, mainly, CO2 and temperature.

In this paper, we argue that the analysis of global warming must depend on identifying the nature of the trend, not only in temperature, but in greenhouse gases data too. If CO2 and temperature exhibit trends, then the moments (the mean, variance, kurtosis, and skewness) are functions of time and therefore, the correlation between these variables is spurious (i.e., meaningless) *in general*, unless these two variables share a common long-run trend (i.e., cointegrated), Granger and Newbold (1974) and Engle and Granger (1987). Predicting the trend is difficult (Phillips, 2003). In this paper, we examine whether the trend is stochastic (i.e., unit root) or linear. If the trend is linear, the time series is said to be trend-stationary, i.e., the trend-adjusted time series is stationary, or I (0,) and its moments are not functions of time. If the trend is stochastic (i.e., unit root), the time series is difference-stationary.[i] However, the correlation between the trended time series is not spurious, if the two series share a common long run trend, i.e., temperature and CO2 are cointegrated. Cointegration means that there is a stationary linear combination of the two trending time series CO2 and temperature.

Another important question that we attempt to answer is whether temperature and CO2 share a common cycle. Common cycle could be conditional on cointegration as in Vahid and Engle (1993). We also examine the correlation between the cyclical components of the time series. We do so by using frequency filters to remove the noise and the trend first, and then test for correlation between the remaining cyclical components.

Furthermore, we test the dynamics of the data by investigating whether it could tell us whether, or not, past information (i.e., the lagged values) of CO2 could explain *current* temperature. We estimate a Vector Auto Regression (VAR). We also measure the effect of CO2 on temperature conditional fossil fuel consumption and world population and how this dynamic affects current temperature, which would be informative for policymakers.

We take the data from NOAA and NASA. We focus on the recent history because CO2 emissions due to industrialization, modern mechanized agriculture, increasing use of fossil fuel, electricity generation, combustion engines, etc., at least in the Northern Hemisphere, increased significantly in the second half of the 20 century. Our data, therefore, are from 1959 to 2020. The data for CO2 measure mean global CO2. The method of measurement is described on the NOAA Website. CO2 is expressed as a mole fraction in dry air, micromol/mol, abbreviated as parts per million (ppm). BP Statistical Review (2020) also reports data for CO2 emission in millions of tones, which looks very similar to the NOAA data. Average global land and ocean temperature (Celsius) data are from the NOAA. NASA compiles similar data, but smoother, and they include Northern and Southern Hemispheres. It is informative to test the Northern and Southern Hemispheres separately because the North should logically have more emissions that the South. The NASA data source is GISTEMP (2021) and Lenssen (2019). We plot the data, which we use in this paper in figure (2). Visually, the data have trends.

We test the data in many different ways; use a variety of methods, models, and specifications for robustness. Here is a summary of our findings. First, we present convincing statistical evidence that the level of CO2 cannot explain the variations in temperature. Second, CO2 is a difference-stationary time series, I (1), i.e., has a unit root, but temperature is trend-stationary, i.e., no unit root as suggested by Chang at al. (2020). We show that the correlation is positive but statistically insignificant. However, if temperature has a unit root then the correlation between the differenced stationary CO2 and the percentage change in temperature is insignificantly different from zero. Third, there is no cointegration – i.e., the levels of CO2 and temperatures do not share a common long run trend. Fourth, we found that there is a statistically insignificant correlation between trend-adjusted (i.e., stationary) global Co2 and trend-adjusted global temperature; trend-adjusted global CO2 and trend-adjusted Northern hemisphere temperature; and trend-adjusted CO2 and trend-adjusted Southern hemisphere temperature. Fifth, the variance of trend-adjusted temperature is 100 times larger than the variance of trend-adjusted CO2. Sixth, there is no evidence of long run co-variation. Seventh, the cyclical fluctuations obtained after removing noise and trend from the data indicate a weak cyclical correlation, 0.53. Eighth, we found that unrestricted (*atheoretical*) VARs present a useful summary of the dynamics of CO2 and temperature, whereby past information of (lagged values) has a significant, however, short-lived, and a small in magnitude, predictive power of *current* temperature, more so in the Northern hemisphere. Ninth, the variance of temperature due to CO2 is between 20 and at most 40 percent; and significantly less for fossil fuel consumption.

Including other greenhouse gasses such as Methane and Sulfur Hexafluoride did not alter the results. We conclude that although many people tend to believe that increasing measured CO2 in the atmosphere is not a good thing, which might very well be true, there is no statistically robust evidence – in our sample of 60 years of the most industrialized times on earth – that it is related to global warming. Policy, therefore, should be evidence-based. *Manmade* greenhouse gasses will be reduced when people realize more profitable investment opportunities in greener economic activities without government intervention based on flimsy statistical evidence.

Next, we briefly discuss the most relevant literature. In section (3), we then test the nature of the trend in CO2 and in temperature. Section (4) answers the question of whether CO2 and temperature share a common long-run trend, i.e., cointegrated. Section (5) examines the cyclical correlations. Section (6) tests for the correlation, and the long-run correlation, between the two variables are in section. In section (7), we look at the short-run dynamics. We estimate a number of unrestricted bivariate VARs, which include CO2 and temperature. Section (8) investigates whether omitted variables might have some explanatory power. We include global fossil fuel consumption in the VARs, and world population growth as an additional exogenous variable. Section (9) is a multivariate analysis of all available greenhouse gases effects on temperature, and section (10) includes a summary and conclusions.

2. Most relevant literature review

The literature includes a number of attempts to examine the nature of the trend in temperature. The evidence about the nature of trend in temperature, and greenhouse gases is mixed. Chang *et al.* (2020) cites the studies that "generate results consistent with unit root in temperature," which are Gordon (1991), Woodward and Gray (1993, 1995), Gordon *et al.* (1996), and Kärner (1996). The studies that "generate results" consistent with the presence of a deterministic trend with possibly highly persistent noise, on the other hand, include, Bloomfield (1992), Bloomfield and Nychka (1992), Baillie and Chung (2002), and Fomby and Vogelsang (2002). Chang *et al.* (2020) found direct support to a one-unit root process (stochastic trend) in the Southern Hemisphere, and two unit root processes in the Northern Hemisphere and the globe, with no evidence for higher-order processes in any of the moments of any of these distributions over time. They acknowledge the difficulty in distinguishing between a stochastic trend and a deterministic trend with breaks using statistical techniques alone.

There are more studies about the trend in temperature, however. Seater (1993), for example, is concerned that there has been no significant trend in temperature, thus, global warming is problematic. He focused on temperature data only, i.e., not CO2. He studied three data sets on world temperature, and argued that "data on direct measurements of world temperature over the past century yield trend estimates of .45 degrees Celsius per century with rather wide confidence intervals of (.15, .75). The data's behavior raises questions about whether the trend is genuine or due to greenhouse-gas emissions." He means by genuine that it is natural and unaffected by CO2 and other emissions. He says, "Data on temperature measurements inferred from tree rings over the past 1,500 years display no trend. The upward drift over the past century could easily be a cyclical upswing of the type that has occurred many times in the past." His method was regressions of the temperature data on a constant term, linear trend, and lagged dependent variable. Indeed, some of the long time series data of temperature, measured from tree rings, have no obvious trend.

Shakun *et al.* (2012) argued that, "The covariation of CO2 concentration and temperature in Antarctic ice-core records suggests a close link between CO2 and climate during the Pleistocene ice ages. The role and relative importance of CO2 in producing these climate changes remains unclear, however, in part because the ice-core deuterium record reflects local rather than global temperature." He constructed a record of global surface temperature from 80 proxy records and showed that, "temperature is correlated with and generally lags CO2 during the last (that is, the most recent) deglaciation. Differences between the respective temperature changes of the Northern Hemisphere and Southern Hemisphere parallel variations in the strength of the Atlantic meridional overturning circulation recorded in marine sediments. These observations, together with transient global climate model simulations, support the conclusion that an antiphased hemispheric temperature response to ocean circulation changes superimposed on globally in-phase warming driven by increasing CO2 concentrations is an explanation for much of the temperature change at the end of the most recent ice age.”

McKitrick (2014) pointed out that, “The Intergovernmental Panel on Climate Change (IPCC) has drawn attention to an apparent leveling-off of globally-averaged temperatures over the past 15 years or so. He argued that measuring the duration of the hiatus (break in the data) has implications for determining if the underlying trend has changed, and for evaluating climate models. Here, I propose a method for estimating the duration of the hiatus that is robust to unknown forms of heteroskedasticity and autocorrelation (HAC) in the temperature series and to cherry picking of endpoints. For the specific case of global average temperatures, I also add the requirement of spatial consistency between hemispheres. The method makes use of the Vogelsang and Frances (2005) HAC-robust trend variance estimator, which is valid as long as the underlying series is trend stationary, which is the case for the data used herein. Application of the method shows that there is now a trendless interval of 19 years duration at the end of the Had CRUT4 surface temperature series, and of 16 - 26 years in the lower troposphere. Use of a simple AR1 trend model suggests a shorter hiatus of 14 - 20 years but is likely unreliable.”

McKitrick and Vogelsang (2014) compare trends across climatic data sets using heteroskedasticity and autocorrelation robust methods, specifically the Vogelsang – Frances (VF) nonparametric testing approach, to allow for a step-change in the mean (level shift) at a known or unknown date. The VF method is robust to unknown serial correlation up to but not including unit roots. They show that the critical values change when the level shift occurs at a known or unknown date. They derive an asymptotic approximation that can be used to simulate critical values, and outline a bootstrap procedure that generates valid critical values and p-values. This method builds on the literature comparing simulated and observed trends in the tropical lower troposphere and mid-troposphere since 1958. The method identifies a shift in observations around 1977, coinciding with the Pacific Climate Shift. Allowing for a level shift causes apparently significant observed trends to become statistically insignificant. Model overestimation of warming is significant whether, or not, a level shift is accounted for, although null rejections are much stronger when the level shift is included.

3. Examining the trend

3.1 . Graphical correlation

The correlation between CO2 and temperature depends on the type of trend in the data. Determining the nature of the trend of both CO2 and Temperature, therefore, is crucial. Figure (2) shows a trend in both, with CO_2 rising smoothly over time while global, Northern, and Southern hemispheres temperatures have trend and fluctuate more than CO_2. Before we test for, or remove, the trend, figure (3) shows the confidence ellipse (a 95% Chi-Squared test) of the correlation between the *levels* of global CO_2 and global temperature. The correlation is positive, as expected, and statistically significant. However, graphical correlation may, or may not, be verified by a regression equation.

3.2 Regression analysis

Table (1) reports six OLS regressions in three panels. The dependent variables are global temperature in panel (1), Northern Hemisphere in panel (2), and Southern hemisphere in panel (3); the explanatory variable in all three panels is CO_2. Each panel has two regressions, without, and with, a constant term. The results are very informative. The regressions with constant terms are significantly different from the ones without. In the first panel and first column, regression, without a constant term, $\boldsymbol{R}^2$ is low equal to 0.15, and the DW statistic is low equal to 0.53. The slope coefficient is 0.001. In the second regression in the first panel, with a constant term, there is a significant negative intercept, $\boldsymbol{R}^2$ increased to 0.77, and the DW statistic increased to 1.92, which is a significantly improved result. This regression seems very reasonable in the terms of increased goodness of fit and serially uncorrelated residuals. The estimator is BLUE.[ii] Moreover, the slope coefficient increased from 0.001 to 0.009. In this regression, one could interpret the results to say that a 100-ppm increase in CO_2 would increase temperature by 0.9 degrees. The regression suggests that, perhaps, there are *missing* variables that explain global temperature, which are captured by the constant term. Scientists must know what these variables are. Nawaz and Sharif (2019) cite Lamb (1997) and they reported, "Who [Lamb] is considered the father of modern climatology, argued that CO2 levels alone couldn't account for all of the global warming that's been observed." Lamb's conclusion seems consistent with our results.

In the second panel, similar things happened when the regression included a constant term, $\boldsymbol{R^2}$increased from 0.14 to 0.88 in the case of the Northern Hemisphere. The slope coefficient increased from 0.001 to 0.013. However, this regression, unlike the global temperature regression, is likely to be spurious because the DW statistic is low. In the third panel, second regression $\boldsymbol{R^2}$ increased to 0.85. The DW statistic was low. This regression is also spurious.

2.2. *Testing the trend*

The difference between trend and difference-stationary time series is that the trend-stationary time series tends to return to a fixed deterministic trend function or it would fluctuate around a fixed trend function. The differenced-stationary time series, however, has no tendency to return to a fixed trend function. It simply grows at a rate β from its current position. Differencing might render the data stationary, most of the time. A stationary time series will be $\mathrm{I}(0)$, i.e., stationary, but not all $\mathrm{I}(0)$

time series are stationary. Some AR (1) model's can be stationary but they are not $I(0)$. Also, not all non-stationary times series are $I(1)$. The inability to determine the nature of the trend results in misspecification with all common consequences such as inconsistency of the coefficients, see for example, Hamilton (1994).

We test the nature of the trend using a number of commonly used unit root tests, e.g., the Dickey - Fuller (1979) – Augmented Dickey-Fuller (Said and Dickey, 1984), the GLS (Elliot, Rothenberg, and Stock, 1996), and Phillips-Perron (1988). One should be cautious about the ability (the power) of these tests to tell the difference between a root of one and 0.98. This is a conclusion shared by many economists, see for example, Stock (1991), Cochrane (1991), Rudebusch (1993), and Christiano and Eichenbaum (1990). There is a large literature about measuring the power of these tests, which we will not cite; however, there is a consensus that the power of these tests is low.

Table (2) reports the ADF test results for global temperature and CO2. The OLS regression specification includes a constant term and a linear trend. For the Augmented Dickey-Fuller test, we use Akaike, Schwarz, and Hannan-Quinn Information Criteria, and the modified versions of them to choose the lag structure.[iii] More lags weaken these tests further. However, the ADF overwhelmingly rejects the null hypothesis of a unit root in temperature. Linear trend, though, is statistically significant. For CO2, the null hypothesis of unit root could not be rejected.

The results of the Phillips – Perron nonparametric test statistic reported in table (3), which are identical to the ADF test results. We use a variety of methods to estimate the spectral density function. All specifications indicate rejection of the null hypothesis of unit root. The linear trend is statistically significant too.[iv]

Table (4) reports the ADF-ERS test results. The test rejects the unit root in temperature when the number of lags in the regression is zero; the test fails to reject the unit root in temperature when the lags increased. This is typical because the power of the test deteriorates fast with more lags. For all of the three test statistics above, even when we fit different models, i.e., with a constant term only, or without a constant term and without a trend, we could not reject the unit root in CO2 and with a much higher P value (1 and close to 1).

Finally, we test for unit root with a breakpoint (in the intercept and the trend) in the temperature data *only* because we suspect that the data have been measured using many different methods over time. The results are in table (5). It included the results of a number of specifications: the ADF test with minimum intercept break t-stat test, maximum intercept t-stat test, and maximum intercept beak absolute t-stat test. In addition, we specify an innovation and additive outliers, and breaks in the intercept and the trend. For each of these specifications, we use the same Information Criteria that we used earlier to determine the lag structure. All tests reject the unit root except when the modified Information Criteria are used to determine the lag structure because the number of lags increased, which weakened the test.

Although the tests that we used to test the trend have low powers, however, when a weak test rejects the null, i.e., in the case of temperature, the power of the test becomes irrelevant. We take the rejection of the null results in the case of the temperature time series to be statistically meaningful and conclude that global land and ocean temperature is *not* a unit root process. However, there is a significant linear trend; hence, temperature is highly probably a trend-stationary time series. We found the same results for NASA's Northern and Southern land and Ocean temperatures data. We do not report these results but they are available on request.

One last test is the KPSS (1992) nonparametric test. It cannot reject the null hypothesis that temperature is I(0) stationary series, with a significant linear trend, hence consistent with all other tests. The value of the KPSS test is 0.071578, i.e., smaller than the 1, 5, and 10 percent critical values that are reported in Kwiatkowski, Phillips, Schmidt & Shin (1992, table 1). The results do not change when we choose different methods to estimate the spectral density. However, because its null hypothesis is I(0) – not I(1) as in the other previous tests; we cannot compare the powers of the tests. Nonetheless, the test confirms that temperature is not a unit root process.

Figures (4a, 4b, 4c,) plot the actual data, their linear trend, and their stationary trend-adjusted temperatures. However, the non-rejection of the unit root by all tests of CO2 indicates that CO2 is a difference-stationary time series.

2.3. The spectral density

To shed more light on the unit root, we also estimated the spectral density of CO2 and global temperature. Figures (5a and 5b) plot the spectral density functions. There is a very clear difference. CO2's spectral is more consistent with a unit root, whereby there is a relatively higher activity at zero frequency, albeit not close to 1, while temperature's spectral is relatively flat, and there is very low activity at zero frequency.

2.4. Descriptive stats of the trend-adjusted data are meaningful

Now since we have assessed the nature of that trend, the moments of the trend-adjusted data are not functions of time and are meaningful. Table (6) reports such descriptive statistics for the trend-adjusted stationary data. Note that the variance of log-difference CO2 is 100 times smaller than the variance of the trend-adjusted temperature. Therefore, variations in CO2 cannot explain the variations in temperature.

4. Do CO2 and temperature share a common long-run trend

4.1 Cointegration

The next testing step in the unit root analysis is to test the null hypothesis that temperature and CO2 are "not" cointegrated, and try to reject it! If the level of CO2 and the level temperature (i.e., not adjusted for trend) share a common long run trend, the two variables are cointegrated. Since temperature and CO2 are non-stationary data, cointegration essentially implies the existence of a stationary, I (0), linear combination of the two variables, i.e., the Triangular Representation Theorem, e.g. Granger (1983).

We test the null hypothesis that CO2 and temperature (the levels) are "not" cointegrated. If we reject this null hypothesis, we may conclude that they share a common trend in the long run. Testing the null of "no" cointegration requires a long *span* of data, and 62 years of annual data is a sufficient span, see for example, Hakkio and Rush (1991). Tables (7a, 7b and 7c) present the results. Typically, the testing is done in steps. In table (7a), we regress the *level* of global temperature on the *level* of CO2 using OLS; and we output the residuals. Engle and Granger (1987) suggested six different ways to test for cointegration. One of them is to test the residuals for unit root using the ADF test. Thus, in table (7b), we test the residuals from the first regression for unit root using the ADF test. The test is distributed Engle-Granger

(1987) and it is best suitable for testing a bivariate system. The weak ADF test cannot reject the unit root with a P value equal 0.9709. Thus, it suggests that the residuals are I(1), hence the two variables are not cointegarted – i.e., do not share a common trend. However, a necessary and sufficient condition for cointegration is the Triangular Representation Theorem, which involves estimating an Error Correction equation. Therefore, in table (7c), we regress the trend-adjusted global temperature on a constant, the log-differenced CO2, and the lagged residuals from the previous *level* regression. For cointegration to exist, the t-statistic on the coefficient of the lagged residuals must be very large (P value is 0). We found that the t-statistics to be statistically insignificant with a P value 0.5853. We conclude that the temperature and CO2 are not cointegrated – i.e., they do not share a common long-run trend and there exist no linear stationary combination of the two variables.

Our results are similar to those published in Phillips *et al.* (2020). They tested temperature (T), CO2 and radiation (R) time series for cointegration, and say, "Table B.1 in Appendix B provides residual based tests for cointegration among the aggregate variables (T, R, CO2). These results are strongly confirmatory of a long run linkage among these three variables taken together but show no direct linkage between the two component variables (R, CO2) or between (T, CO2). This confirms the role that R and CO2 play jointly in the long run determination of T."

4.2 The long run covariance

Alternatively, we check whether there is a long-run co-variation between the trend-adjusted CO2 and temperature. Table (8) reports the long-run covariance between CO2 and temperature. These long-run co-variances are symmetric, degree-of-freedom adjusted, the weights are chosen using Akaike Information Criterion (AIC), the kernel is computed using Bartlett method, and the bandwidth method is the Newey-West. The long-run covariance is close to zero.

5. Do CO2 and temperature have common cycle

Because we found no cointegrating vector between CO2 and temperature, we cannot use Vahid and Engle (1993) to test for a common cycle, which is based on squared canonical correlation and conditional on the number of cointegrating vectors. However, we examine the cyclical relationship, which Seater (1993) suspected by decomposing the time series into trend, cycle, and noise using symmetric and asymmetric Band Pass frequency Filter, Christiano – Fitzgerald (1995). The typical cycle periodicity in an annual data is 2 to 8 years. Figures (6), (7), (8), and (9) are the cyclical fluctuations of the average global temperature, the average Northern Hemisphere temperature, the average Southern Hemisphere temperature, and CO2. Figure (10) plots together the cyclical fluctuations of global temperature and CO2. There is a weak 0.53 correlation between them over the cycle.

6. Correlation

Since we have no statistically significant evidence of long run and cyclical relationships between CO2 and temperature, we examine the correlation between the trend-adjusted data. Figures (11), (12), and (13), plot the Chi-Squared 95% Confidence Ellipses, which tests the significance of the correlations between the trend-adjusted CO2 (differenced stationary) and temperature (trend stationary). These tests show that there is a positive but statistically insignificant correlation between CO2 and temperature globally.

It is important to mention that if temperature is assumed to be a unit root processes as suggested by Chang *et al.* (2020), then the correlation between the differenced stationary temperature and the differenced stationary CO2 is, in fact, zero. Figures (11b), (12b), and (13b) plot the Chi-squared tests.

7. The short-run dynamics: could past CO2 information predict current temperature?

So far, there are no significant short-run, long run, or cyclical relationships between temperature and CO2. Here, we examine the dynamic, i.e., whether past information of CO2 (i.e., lagged values) has any predictable power of *current* temperature. We summarize the dynamics of the data using an unrestricted Vector Autoregression

(VAR). Essentially, a bi-variate unrestricted VAR is similar to the so called Granger causality test, where by the variables are regressed on their own lagged values and the lags of the other variables, then the null hypothesis that X does not Granger-cause Y and Y does not Granger-cause X are tested using an F statistic.

Our *atheoretical* VAR is unrestricted, i.e., we do not impose theoretical restrictions on the VAR, because the econometrician does not have theoretical restrictions to impose on the variables to identify the shocks. The model is not an economic model, and the theory about the relationship between CO2 and temperature is not an economic theory. Therefore, we will simply examine the dynamic effect of CO2 on temperature. We view this VAR as a method to summarize the dynamics of the two variables, CO2 and temperature, no more than that.

The VAR is:

$$y_t = A_1 y_{t-1} \cdots A_p y_{t-p} + C x_t + \varepsilon_t \,, \qquad (1)$$

where $y_t = (y_1, y_{2t}, \cdots y_{kt})'$ is a $k \times 1$ vector of endogenous variables, and $x_t = (x_1, x_2, \cdots x_{dt})'$ is a $d \times 1$ vector of exogenous variables. $A_1 \cdots A_p$ is $k \times k$ matrix of lag coefficients , and C is a $k \times d$ matrix of the exogenous variables' coefficients.

There is also an exogenous constant term, $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \cdots \varepsilon_{kt})'$ is a $k \times 1$ vector of white-noise innovations with $(\varepsilon_t) = 0$; $E(\varepsilon_t \varepsilon_t') = \sum \varepsilon$, and $E(\varepsilon_t \varepsilon_s') = 0$ for $t \neq s$.

We begin with an unrestricted VAR without any exogenous variables. We examine the growth rate of CO2 and trend-adjusted global temperature first. We test the number of lags using a number of commonly used exclusion tests (sequential modified LR test statistic, final prediction error, AIC, SIC, and HQ criteria). We chose two lags because temperature is volatile, which affects the calculation of the variances. We think that variance decomposition is more informative than impulse response function in this case. The variance decompositions (standard errors are generated using 1000 Monte Carlo iterations) are plotted in figure (14). The variance of the trend-adjusted global temperature due to CO2 growth is no more than 20 percent. A structural VAR does not alter the results.[v]

Then we estimate the same VARs for $\Delta\, ln$CO2 and the trend-adjusted Northern and Southern Hemispheres trend-adjusted stationary temperatures. Figure (15) shows that the variance of the Northern Hemisphere temperature due to CO2 growth is about 30 percent. Figure (16) shows a similar result for the variance of the Southern Hemisphere due to CO2 growth. Figure (17) puts the variance of the three measures of temperature due to CO2 growth together. These *short-run* dynamics indicate that past information of CO2 could predict a small percentage of the current temperature over the sample from 1959 to 2020.[vi]

8. An Omitted variable problem

It is quite reasonable to assume that there are other variables, which might affect both temperature and CO2, i.e., omitted variables. We had this clue from the regressions in Table (1). The constant terms were very significant, and they changed the goodness of fit of the regressions. There are potential effects of solar variation, cosmic ray flux, and the Milankovich cycles, and soil erosion and desertification, which may explain some of the unexplained variations in temperature. This would the scientists' job. In this paper, however, we are more concerned with the effect of the greenhouse effect on policy. Given that non-fossil fuel – non-manmade greenhouse gasses can also affect global warming, we want to test the effect of CO2 on temperature, conditional on fossil fuels, production and consumption, which were missing from our previous VAR, and they could be affecting the dynamic. In another word, we test whether adding a third variable that affects CO2 might change the relationship between with temperature.[vii]

We estimate a VAR, which includes in addition to the growth rate of CO2 and trend-adjusted global land and ocean temperature, the growth rate of global fossil fuel consumption. Fossil fuel consumption is the sum of oil, gas, and coal consumptions measured in Exajoules (BP Statistical Review, 2020). We also included in the VAR the world population growth as an exogenous variable. World population (OECD Statistics) has been growing over the past sixty years, and it could have some significant exogenous effect on greenhouse gasses.

Figure (18) plots the additional variables, global fossil fuel consumption, and world population. Both have unit roots. Figure (19a), (19b) and (19c) report the variance decompositions, i.e., percent temperature variance due to CO2, based on Cholesky degree-of-freedom adjusted, and 1000 Monte Carlo generated standard error. The growth rate of CO2 explains more of the variance of the Northern Hemisphere temperature than it does for the Southern Hemisphere, and global temperatures, about 40 percent. It explains just a little more than 20 percent of the global temperature, and about 30 percent of the Southern Hemisphere's. Figures (20a), (20b), and (20c) plot the variances of temperature due to fossil fuel. The amount of variations in temperature due fossil fuel consumption growth is trivial.
Scientists believe there is a feedback mechanism that significantly influences the CO2 – temperature connection. Perhaps they reached the same conclusion Lamb (1997) has reached, which is consistent with our results in table (1). See the Royal Society on water vapor feedback effect on temperature. Dlugokencky (2016) *et al.* say, "This strong water vapor feedback means that for a scenario considering a doubling of the CO_2 concentration from pre-industrial conditions, water vapor and

clouds globally lead to an increase in thermal energy that is about three times that of the long-lived greenhouse gases. Therefore, measured in the ability to trap the heat emanating from the Earth's surface, water vapor and clouds are the largest contributors to warming. The amount of water vapor in the atmosphere is a direct response to the amount of CO_2 and the other long-lived greenhouse gases, increasing as they do."

Note that there are many different methods to measure water vapor. At least eight are mentioned in Dlugokencky (2016) *et al.* However, these methods make it difficult to have a consistent trend measurement. They argue that "for example, the limited lifespan of satellite missions or insufficiently documented or understood changes in instrumentation. Combining records from different instruments that do not agree with one another is also a problem. One example is the offset between records from the HALOE and MLS satellite instruments. Nevertheless, observations show a steady increase of the total water vapor column as well as a 30-year net increase in stratospheric water vapor. Also, see Ning et al. (2016), for example, who found number of breaks in the integrated water vapor (IWV) time series obtained from reprocessed data acquired from global navigation satellite systems (GNSS). Furthermore, it is very difficult to get a consistent time series data for water vapor online. We conclude that since water vapor amplifies CO2 effect on temperature, our Dynamic OLS regressions include lag and lead CO2, albeit one lag and one lead, they should capture some of the water vapor feedback effect. The data for water vapor are not readily and easily accessible from NOAA webpage, and there is no time series data as far as we know.

9. The other greenhouse gases

Finally, we attempt to include the other greenhouse gases in our analysis. NOAA publishes data for methane, CH_4, Sulfur Hexafluoride SF_6, and Nitrous Oxide,N_2O. However, they come with different samples. Take methane (CH_4) for example, the sample is 1984 to 2019. SF_6has a shorter sample from 1998 and N_2O is very short from 2001. The sample size affects the time series analysis, and the method of analysis. We use a multivariate method for cointegration instead of bivariate methods. Water vapor is also a greenhouse gas.

We begin with the three gases, methane, Nitrous Oxide, and Sulfur Hexafluoride because some time series data are readily accessible. We test for unit root just like did before. Methane (CH4) has a unit root because the common tests for unit root could not reject the null, which might be due to low power. Nonetheless, it has a significant positive trend. Thus, it is differenced-stationary. For Sulfur Hexafluoride SF6, has a unit root too. Nitrous Oxide,N_2O data are unsuitable for time series analysis. Figure (21) plot these time series.

To test the null hypothesis of "no" cointegration, we follow two strategies. First, we test pair wise, with temperature exactly like what we have done with CO2 earlier and use the bivariate Engle-Granger (1987) method. So, we test temperature and CH_4 then we test temperature and SF_6 separately.

Table (9) reports the results of the temperature-Methane pair. In (9a), we regress global temperature on a constant and Methane. In table (9b), we test the residuals from the above regression for unit root using the ADF test. The test with the AIC rejects the unit root and with the modified, AIC cannot reject the unit root, however, in (9c), the lagged residuals are statistically insignificant from zero, which suggest that there is no evidence of cointegration between global temperature and methane. This latter ECM regression is a more reliable method to test for the null hypothesis of no cointegration. Table (10) tests the pair of temperature-SF_6. The sample is much smaller, from 2001to 2019. There is no evidence of cointegration here either.

Since none of the greenhouse gasses is cointegrated with temperature, we do not expect a multivariate test for temperature and all the three greenhouse gases together to provide any insight about the long run common trend between them. It would indicate cointegration, but it would be a cointegration among the greenhouse gasses themselves, and not with temperature. The Johansen Maximum Likelihood test suggests two to three cointegration relationships. Two cointegration relationships are probably between CO2 and CH_4 because the sample is longer while the sample of SF_6 is short. These multivariate results are also similar to Phillips *et al.*

(2020). They did not find a cointegration relationship between CO2 and temperature, but they found a cointegration relationship among CO2, Temperature, and radiation.

Table (11) reports the results. We test the null hypothesis of no cointegration among the four variables temperature, CO2, CH4, and SF6 using both the bivariate Engle-Granger (1987) and the Johansen's Maximum Likelihood Test, Johansen (1988, 1991 and 1995) and Johansen and Juselius (1990). The Johansen tests in more appropriate in a multivariate case like this one. However, the sample size is short, 22 years as compared with 62 in the previous analysis because the sample for SF_6 is short, 2001 to 2019. Cointegration requires a long span of data, and the Johansen test statistics, i.e., the Trace and the Maximum Eigenvalue, have a small sample bias that is very difficult to fix, therefore, one should take these results with a grain of salt. See, for example, Cheung and Lai (1993) for correcting the critical values of the Johansen's test statistics.

For the Engle-Granger test, we use OLS to regress global temperature on the levels of CO2, CH_4, $SF_6$and deterministic linear trend; output the residuals; test them for unit root using the ADF test; and finally estimate an OLS error correction equation. The tests suggest that these variables are cointegrated. The error correction term has a large t statistic (p value is 0.0033). The ADF also strongly rejects the unit root in the residuals of the level regression. The Johansen tests include intercept and trend in the cointegration equation and no intercept in the VAR. This is the only plausible specification because we tested the normalized cointegration relationships for unit root using the ADF and we could reject the unit root at the 10 percent level. Recall that the Johansen test statistic and the ADF are identical in the case of one unit root. Other specifications do not seem to be same.[viii]

Figure (22) plots the three-cointegration relationships and the residuals from the level regression of temperature on the greenhouse gasses, i.e., the Engle-Granger cointegartion relationship. These plots suggest that there might be a long trend among these variables albeit one should be careful about such interpretation because of the small sample problem. The first cointegration relationship is indeed I(0), the second too, but the third has a trend. The Engle-Granger residuals are I(0). A cointegration relationship among the greenhouse gasses is not a surprise in general. The question is whether the share one with temperature.

Cointegration implies that we could either run OLS regressions in levels, whereby the t statistics is still valid or we could use other error correction methods such as VECM, FMOLS and Dynamic OLS. We do both. In table (12), we report several regression results. All variables are in *levels*. The dependent variable is temperature, and the regressers are CO2, CH_4, and SF_6. The table has six panels. The first three panels are for OLS. The third panel is Dynamic OLS (see Phillips – Loreatn (1991), Saikkonen 1991, and Stock and Watson (1993) for the asymptotic theory).[ix] Each panel has two columns. The first column reports regressions without a constant term. The second column reports the regressions with constant terms.

The first three panels are the OLS results for average global land and ocean temperature, the Northern Hemisphere temperature, and the Southern Hemisphere temperature respectively. The regressions without constants suggest that all greenhouse gasses are statistically insignificant explanatory variables for temperature. The goodness of fit is low in global temperature regression in column (1), 0.29 and DW statistic is 1.94. Adding a constant to the regression in the second column makes all coefficients statistically significant and improves the fit, adjusted R-squared increases to 0.50, not particularly high. The DW statistic is 2.3. CO2 has a large coefficient 0.25; methane coefficient is 0.023 and SF_6 is negative 2.30. The point is that the constant term captures more missing explanatory variables. Nevertheless, most importantly, these estimates are nonsensical. A 100-ppm increase in CO2 raises global temperature by 25 degrees Celsius!

In the second panel for the Northern Hemisphere temperature, the first column reports all three explanatory variables are statistically insignificant. The relatively high adjusted R-squared and low DW statistic suggests that the regression is spurious. Again, the results change when we add a constant to the regression. The coefficients are statistically significant and the errors are serially uncorrelated with a

DW statistic 1.72. Here too, the coefficient of CO2 is 0.15, too large to make any sense.

And, in the third panel for the Southern Hemisphere temperature, the first column is the regression without a constant, and the coefficients are insignificant except for SF_6. The goodness of fit, adjusted R-squared is 0.63 and the DW statistic is 1.78, however, surprisingly when we add a constant to the regression the coefficients become statistically insignificant. In all regressions without a constant CO2 has a negative sign. We believe that the short sample size has some effect on these results and one should be careful interpreting them. That said, the constant term must be accounting for some other explanatory variables.

The last panels report the results of the Phillips – Loreatn (1991) dynamic-OLS in *levels* to estimate the coefficients. The small sample size restricted our ability to search for the optimal lag length. Therefore, we fixed the lag-lead length to be one lag. None of the coefficients is significant, except forCH_4, methane. The regressions of the global temperature in column (g) and (h), without and with a constant, have insignificant coefficient estimates, except for methane, which has a coefficient of 0.03 and statistically significant. The regression in column (i) has all the coefficient estimates insignificant. Adding a constant term to it in column (j) makes all the coefficients significant, improves the adjusted R-squared to 0.88, but the magnitude of CO2 coefficients is too large to make sense, 0.31. For the Southern Hemisphere temperature, both regressions in column (k) and (l), without and with a constant term are insignificant.

The results in table (12) are not robust to specifications and methods. The sample size is too small perhaps. The Phillips-Loretan Dynamic OLS is restricted to one lag/lead because of the small sample and that might have affected the estimates. One thing remains clear; a constant in the regression makes a difference and is likely to be telling us that there are any other explanatory variables missing.

Finally, we examine the short-run dynamics by estimating a VAR, which has the variables in this order $\Delta\ lnSF_6$; $\Delta\ lnCH_4$; $\Delta\ ln$CO2 and trend-adjusted global temperature. The sample is 1998 – 2019 because the first variable is only available from 1998 to 2019. Table (13) reports the variance decomposition. These are based on Cholesky, degree-of-freedom adjusted, and 1000 Monte Carlo generated standard error. The variance of global temperature due to $SF_6$is negligible, only slightly more is due to methaneCH_4, but now more variations in temperature is attributed to CO2 than in the previous VAR, about 55 percent. Keep in mind that this VAR has a much shorter sample. The order of the variables did not seem to influence the outcome.

We re-estimate the VAR with the growth rate of fossil fuel consumption in addition to the four other greenhouse variables. The VAR is ordered as follows, growth rate of fossil fuel consumption, followed by $\Delta\ lnSF_6$; $\Delta\ lnCH_4$; $\Delta\ ln$CO2 and trend-adjusted global temperature. The results change significantly because of the inclusion of fossil fuel consumption growth. They might have affected by the short sample too. Table (14) reports the variance decomposition, with the variance of temperature due to CO2 now dropped to less than 10 percent of total variation. More than 10 percent is due to methane, and more than 30 percent is due to fossil fuel consumption. Greenhouse gasses do not seem to explain much of the dynamic of global temperature.

Tables (15) and (16) report the variance decompositions of the Northern and Southern hemisphere temperatures. The variance of temperature due to CO2, however, remains small, but it doubled in the South compared with the North. The variance of temperature due to fossil fuel consumption growth tripled in the Southern hemisphere compared with the North. These results are significantly different from what have seen before adding the other two greenhouse gasses CH_4 andSF_6, and make less sense because we expect more influence in the Northern hemisphere than the South. This dynamic must be influenced by the short sample we have now and the loss of the degrees of freedom to estimate the dynamics.

10. Summary and conclusions

There is a global acceptance among people that CO2 and global temperature are correlated. Some people suggest causation from the former to the latter. It is rather very difficult to argue otherwise. Greenhouse effects, global warming, and climate change are very important contemporary issues to governments, businesses, and people. There is a pressure on governments to adopt policies to reduce or eliminate CO2, fossil fuel, and other greenhouse gasses, which is a costly endeavor. The International Renewable Energy Agency (IRENA) estimates to achieve a zero carbon world and to keep global temperature at 1.5 Celsius by 2050 are huge. It says, “Major economies have announced economic stimulus packages that will pump approximately USD 4.6 trillion directly into carbon-relevant sectors such as agriculture, industry, waste, energy and transport, but less than USD 1.8 trillion is green.” Stimulus is either tax-financed or borrowing, which could have significant economic consequences. Then it goes on saying, “By contrast, energy transition investment will have to increase by 30% over planned investment to a total of USD 131 trillion between now and 2050, corresponding to USD 4.4 trillion on average every year. Socio-economic benefits will be massive; investing in transition will create close to three times more jobs than fossil fuels, for each million dollars of spending. To address concerns about a fair and just transition, IRENA’s Outlook calls for a holistic and consistent overall policy framework.”

Most scientists agree that policy should be evidence-based. The first step is to make sure that CO2 and temperature are correlated in a statistical sense, and that the correlation is stable over time, which is something that seems to have been assumed and taken for granted. The objective of this paper is simple. We test the statistical significance of the association, e.g., correlation, long run common trend, long run co variation, common cycles, etc., between CO2 and temperature.

We began by showing Jouzel, J. *et al.* (2007) graphical correlation between $CO_2$and global temperature over a period of 800,000 years. On the X-axis, age, which is years before present, and on the main vertical axis is temperature, and on the RHS vertical axis is CO2. Most importantly, both variables are in *levels*. So, 800,000 years ago, 500,000 years ago…etc. The correlation between these two variables is remarkable. We thought that this graph would be a scientifically sufficient proof for the existence of correlation between CO2 and temperature. However, given the significant correlation between CO2 and temperature for 800,000 years, obviously not all greenhouse gasses have been *manmade*, not even 30,000 years ago. Second, the relationship shows rising and falling CO2 and temperature 100,000 years ago and now look the same, even less now. Third, there are prolonged periods of low CO2 and low temperature levels, so what caused CO2 level to fall without any interventionist policy? However, most importantly for this paper is that both CO2 and temperature are measured in *levels and have trends.* Trend in the data could render the correlation between the levels spurious. This paper focuses on the time series analysis of trend, and on how to calculate and estimate meaningful associations between CO2 and temperature.

We used a shorter sample that covers a more realistic period of greenhouse gas, which is more appropriate to examine for policy purposes. NOAA and NASA report data from 1959 to 2020, a sample more logical to investigate. This period includes increased industrialization, at least in the Northern Hemisphere, global population growth, marked increases in fossil fuel production and consumption, vast mechanized agriculture, more power generation, more cars, more planes, and many other harmful practices that exploded during the past 60 years. Crippa. M. Solazzo, E., Guizzardi, D. *et al.* (2021) show that "food systems" were responsible for 34 percent of all human caused greenhouse gas emissions in 2015.

In our sample, we can visually identify positive trends in CO2 and temperature, globally and in Northern and Southern hemispheres. Any correlation between the *levels* of these variables is, therefore, spurious, unless they are cointegrated. All the moments are functions of time, hence, uninformative unless, these two variables are cointegrated (i.e., share a common long-run trend). In order to make sense of the relationship between CO2 and temperature, we have to, first, identify the nature of the trend, and second, remove it.

We run a number of OLS regressions of temperature (global, Northern Hemisphere, and Southern Hemisphere) on CO2 from 1959 to 2020 in the levels. In global temperature, we found no correlation whatsoever, and the residuals were significantly serially correlated. Then we repeated the regressions by adding a constant term. This regression changed significantly, the fit increased significantly and the residuals became white noise. This suggests that the constant term is accounting for something missing that explains temperature. In the Northern and Southern Hemispheres, the fit also increased significantly, but the residuals remained serially correlated, which is a sure sign of spurious regression.

Since we have sufficient evidence that the trend in the data affects the correlation between temperature and CO2, we investigated the nature of the trend in the data. The trend is either linear or stochastic. We used a variety of commonly used and well-know statistical tests to test for unit root (i.e., stochastic trend) conditional on linear time trend. When testing, we used different specifications, and many different Information Criteria to determine the lag length. At the end, the results are quite significant. We rejected the unit root hypothesis in temperature, but not in CO2. The rejection of the unit root in temperature by already known weak statistical tests is a significant result because the power of the test in this case is irrelevant, i.e., the null has been rejected already. Therefore, CO2 is probably a unit root process, hence, it is difference-stationary but temperature is trend-stationary. As a matter of fact the association between temperature and CO2 completely disappears if we assume that temperature is difference-stationary too.

We proceeded with these conclusions to investigate the correlation between the trend-adjusted data. First, we found weak positive correlation between the two variables as indicated by the Chi-squared 95% confidence interval. Second, the variance of the log-differenced CO2 is 100 times smaller than the variance of the trend-adjusted temperature, thus CO2 could not possibly explain temperature. Third, widely used nonparametric methods indicate no long-run significant covariance in these data. Fourth, a series of tests for a bivariate cointegration, i.e., whether CO2 and temperature share a common long-run trend, indicated that we could not reject the hypothesis that there is "no cointegration". Temperature and CO2 do not share a long-run common trend. Phillips *et al.* (2020) found a similar result essentially. Fifth, because there is no statistical evidence of a cointegrating vector, we fail to test for common cycles. However, sixth, we decomposed the time series into noise, trend, and cycles, removed the noise and the trend and examined the correlation between the cyclical CO2 and cyclical temperature. We computed the correlation to be 0.53, which is very small indeed. Seventh, our final exercise involved estimating a number of unrestricted VARs to examine whether there is a dynamic relationship between CO2 and temperature. In other words, we examine whether past information in CO2 has predictive power of current temperature.

We found that temperature responds positively to CO2 shock. The variance of temperature due to the growth in CO2 is small, less than 20 percent in global temperature, about 30 percent in the Northern hemisphere and the Southern hemisphere. To the extent that such VAR is too small and omitted variables might play a significant role in explaining the dynamic of temperature in such *atheoretical* exercise, we also estimated VARs that include global fossil fuel consumption in addition to CO_2 and temperature. We added population growth is an exogenous variable too. No significant change in the results that we obtained earlier from smaller VARs is found. Most importantly, only up to about 40 percent of the variations in Northern hemisphere temperature is due to CO_2, less than 30 percent in Southern hemisphere, and less than 20 percent in global temperature. For fossil fuel consumption, less than 20 percent of the variations in temperature are due to fossil fuel consumption.

These stylized statistical facts are inconsistent with widely held views and openly expressed opinions of scientific communities, governments, and people at large. The temperature's variance is 100 times larger than the variance of the supposedly explanatory variable, CO_2. Thus, it cannot explain the variation in temperature.

We attempted to examine the effects of the other greenhouse gasses, such as methane, Oxide, and Sulfur Hexafluoride along with CO_2, on temperature in a multivariate analysis. Unfortunately, we do not find anything significant.

Our results are consistent across the various tests that we ran in this paper. The variation in temperature cannot be explained by that of CO_2, and there is a lot of explaining that needs to be done before policymakers take actions. Our results were derived from straightforward statistical methods, which are easily reproducible. The sources of the data we used are referenced and the data are readily available online. We also put the data we used in the appendix.

That said, policy based on the weak statistical relationship between trend-adjusted CO2 and temperature is a questionable policy. However, we believe that it is probably prudent for policy to provide incentives to invest in greener energy, where profit opportunities are available. Indeed, data show that investment in non fossil fuel has been increasing worldwide, see Bloomberg NEF Clean Energy Investment Trends (2020).

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Table (1)
OLS Regression
$Temp_t = \alpha + \beta CO_{2t} + u_t$
Sample 1959-2020

	1	2	3	4	5	6
	Global Temp(i)		Northern Hem Temp(ii)		Southern Hem Temp(iii)	
α	-	-3.14	-	-4.5	-	-2.4
	-	(0.0000)	-	(0.0000)	-	(0.0000)
β	0.001	0.009	0.001	0.013	0.0008	0.007
	(0.8486)	(0.0000)	(0.4973)	(0.0000)	(0.3145)	(0.0000)
R^2	0.15	0.77	0.15	0.88	0.18	0.85
DW	0.53	1.92	0.16	1.22	0.22	1.28
Correlation	Weak	Strong	Weak	Spurious	Weak	Spurious

(i) HAC Standard errors & covariance, Bartlett Kernel, prewhitening with lag =3, from AIC maximum lag=3, Newey-West fixed bandwidth=4
(ii) HAC Standard errors & covariance, Bartlett Kernel, prewhitening with lag =2, from AIC maximum lag=3, Newey-West fixed bandwidth=4
(iii) HAC Standard errors & covariance, Bartlett Kernel, prewhitening with lag =1, from AIC maximum lag=3, Newey-West fixed bandwidth=4
(iv) P values are in parentheses.

Table (2)

$$\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{i=1}^{n} \lambda_i \Delta y_{t-i} + \varepsilon_t$$

H_0:Unit Root

Augmented Dickey – Fuller

Estimator: OLS

Standard errors are HAC – Newey-West with df Adj.

	y_t: Temperature							
	Information Criteria				Modified Information Criteria			
	Lag	AIC	SIC	HQ	Lag	AIC	SIC	HQ
α	0	-0.12	=	=	3	-0.12	=	=
		(0.0104)				(0.0418)		
β		0.015	=	=		0.012	=	=
		(0.0000)				(0.0000)		
ρ		-0.92	=	=		-0.67	=	=
		(0.0000)				(0.0062)		
$\bar{R}^2$		0.44	=	=			=	=
σ		0.16	=	=			=	=
DW		2.00				1.98		
Unit Root		NO	NO	NO		NO	NO	NO

	y_t: CO2							
	Information Criteria				Modified Information Criteria			
	Lag	AIC	SIC	HQ	Lag	AIC	SIC	HQ
α	2	0.41	=	=	0	0.57	=	=
		(0.9391)				(0.9070)		
β		0.03	=	=		0.03	=	=
		(0.2421)				(0.2816)		
ρ		0.001	=	=		0.0005	=	=
		(0.9283)				(0.9760)		
$\bar{R}^2$		0.54	=	=		0.55	=	=
σ		0.45	=	=		0.45	=	=
DW		1.93	=	=		2.04	=	=
Unit Root		YES	YES	YES		YES	YES	YES

Augmented Dickey-Fuller is Said – Dickey test statistic. P values are in parentheses. = means that the results are the same.

Table (3)

$$\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{i=1}^{n} \lambda_i \Delta y_{t-i} + \varepsilon_t$$

H_0:Unit Root

Phillips – Perron

Standard errors are HAC – Newey-West with d.f. Adj.

y_t: Temperature

	Spectral Estimation Method					
	Bartlett and Parzen	Quadratic	AR (OLS)	AR (OLS de trended)		AR (GLS de-trended)
α	-0.12 (0.0104)	=	=	=	=	=
β	0.014 (0.0000)	=	=	=	=	=
ρ	-0.92 (0.0000)	=	=	=	=	=
$\bar{R}^2$	0.44	=	=	=	=	=
σ	0.16	=	=	=	=	=
DW	1.89					
Band Width/Lag	4	8	4.38	/0	/0	/0
Unit Root	NO	NO	NO	NO	NO	NO

y_t: $CO2$

	Spectral Estimation Method					
	Bartlett and Parzen	Quadratic		AR (OLS)	AR (OLS de trended)	AR (GLS de-trended)
α	0.57 (0.9070)	=	=	=	=	=

β	0.03 (0.2816)	=	=	=	=	=
ρ	0.0004 (0.9760)	=	=	=	=	=
$\bar{R}^2$	0.55	=	=	=	=	=
σ	0.45	=	=	=	=	=
DW	2.04	=	=	=	=	=
Band Width/Lag	2	3	1.79	/0	/3	/7
Unit Root	YES	YES	YES	YES	YES	YES

P values are in parentheses. For AR (OLS), AR (OLS detrended) and AR (GLS deterended) the Information Criteria are AIC, SIC, and HQ and the results do not change.

Table (4)
Elliott – Rothenberg – Stock (GLS – ADF Test)

Temperature

	Information Criteria			Modified Information Criteria		
	AIC	SIC	HQ	AIC	SIC	HQ
ERS	-2.26	-5.99	-2.26	-1.18	-1.18	-1.18
	(2)	(0)	(2)	(3)	(3)	(3)
Unit Root	YES	No	YES	YES	YES	YES

CO2

	Information Criteria			Modified Information Criteria		
	AIC	SIC	HQ	AIC	SIC	HQ
ERS	-1.08	-0.79	-0.79	-0.79	-0.56	-0.79
	(7)	(4)	(4)	(4)	(3)	(4)
Unit Root	YES	YES	YES	YES	YES	YES

We test at the 5% level, where the critical value is -3.16120 – Elliott-Rothenberg-Stock (1996) table (1). Parentheses include the number of lags. The estimator is GLS.

Table (5a)

Testing for Unit Root with Break in the Intercept and the Trend

Temperature	Information Criteria			Modified Information Criteria		
Test	AIC	SIC	HQ	AIC	SIC	HQ
Min ADF	-8.15	-8.15	-8.15	-7.9	-7.9	-7.9
	[<0.01]	[<0.01]	[<0.01]	[<0.01]	[<0.01]	[<0.01]
	(0)	(0)	(0)	(0)	(0)	(0)
Break	2007	2007	2007	2010	2010	2010
Type	innovation	innovation	innovation	Innovation	innovation	innovation
Unit Root	No	No	No	No	No	No

Lag length in parentheses, P values are Vogelsang (1993) asymmetric on-sided in squared brackets, no change in results when the break type is an additive outlier. The trend is significant.

Table (5b)

Temperature	Information Criteria			Modified Information Criteria		
Test	AIC	SIC	HQ	AIC	SIC	HQ
Min t-trend	-6.88	*	*	-1.73	*	*
Break t-stat	[<0.01]	*	*	[0.8819]	*	*
	(0)	*	*	(3)	*	*
Break	1985	*	*	1982	*	*
Type	innovation	*	*	Innovation	*	*
Unit Root	No	No	No	Yes	Yes	Yes

Lag length is in parentheses. Asterisk means the results do not change. Vogelsang (1993) asymptotic one-sided P values are in squared brackets, no change in results when the break type is an additive outlier. Number of lags reduced the power of the test, hence increases the chance of non-rejection of the null hypothesis. The trend is significant.

Table (5c)

Temperature	Information Criteria			Modified Information Criteria		
Test	AIC	SIC	HQ	AIC	SIC	HQ
Max t-trend	-8.30	*	*	-2.43	*	*
Break t-stat	[<0.01]	*	*	[0.6688]	*	*
	(0)	*	*	(4)	*	*
Break	2007	*	*	1982	*	*
Type	Additive	*	*	Additive	*	*
Unit Root	No	No	No	Yes	Yes	Yes

Lag length is in parentheses. Asterisk means the results do not change. Vogelsang (1993) asymmetric one-sided P values are in squared brackets, no change in results when the break type is an additive outlier. Number of lags reduced the power of the test; hence increase the chance of non-rejection of the null hypothesis. The trend is always statistically significant.

Table (6)
Sample: 1959 2020
Trend-Adjusted Stationary Data

	CO2	Temp	Northern Hemp Temp	Southern Hemp Temp
Mean	0.004439	-0.004456	0.007033	0.010459
Median	0.004623	-0.012650	-0.002000	0.006000
Maximum	0.008443	0.341050	0.373000	0.178000
Minimum	0.001313	-0.383940	-0.307000	-0.190000
Std. Dev.	**0.001640**	**0.163817**	**0.169711**	**0.085500**
Skewness	0.127377	-0.027184	0.255654	-0.016708
Kurtosis	2.580262	2.629405	2.570077	2.238257
Jarque-Bera	0.612744	0.356587	1.134272	1.477646
Probability	0.736113	0.836697	0.567148	0.477676
Sum	0.270767	-0.271790	0.429000	0.638000
Sum Sq. Dev.	0.000161	1.610161	1.728114	0.438611
Observations	61	61	61	61

Table (7a)

OLS regressions

$$Temp_t = \alpha + \beta\, CO_{2t} + \psi_t$$

$Global\ Temperature\ (NOAA)$

Variable	Coefficient	Std. Error	t-Statistic	P value
α	-3.15	0.30	-10.1	0.0000
β	0.01	0.0008	11.4	0.0000

Adjusted R-squared	0.76	S.D. dependent var.	0.33
S.E. of regression	0.16	Akaike info criterion	-0.81
Sum squared res.	1.50	Schwarz criterion	-0.74
Log likelihood	27.20	Hannan-Quinn criterion	-0.78
F-statistic	199.6	Durbin-Watson stat	1.92
Prob. (F-statistic)	0.000	Wald F-statistic	129.2
Prob. (Wald F-statistic)	0.000		

HAC standard errors & covariance (Prewhitening with lags = 0 from AIC Maximum lags = 3, Bartlett kernel, Newey-West fixed bandwidth = 4.

$$\Delta\psi_t = \theta + \rho\psi_{t-1} + \sum_{i=1}^{n} \lambda_i \Delta\psi_{t-i} + \upsilon_t$$

Lag Length: 3 (Automatic - based on AIC, maximum lag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.207819	0.9709
Test critical values:	5% level	-2.912631	

*MacKinnon (1996) one-sided p-values. Modified AIC gives a similar result.

Table (7c)

Error Correction

$$\Delta Temp_t = \delta\, \Delta \ln CO_{2t} + \pi\, \psi_{t-1} + u_t$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.17	0.083	-2.034	0.0466
δ	36.57	15.913	2.30	0.0252
π	-0.046	0.0843	-0.55	0.5853

R-squared	0.11	Mean dependent variable	-0.004
Adjusted R-squared	0.08	S.D. dependent variable	0.16
S.E. of regression	0.15	Akaike info criterion	-0.82
Sum squared res.	1.42	Schwarz criterion	-0.71
Log likelihood	28.0	Hannan-Quinn criterion	-0.78
F-statistic	3.74	Durbin-Watson stat	1.57
Prob. (F-statistic)	0.029	Wald F-statistic	2.69
Prob. (Wald F-statistic)	0.076		

$\Delta Temp$ is trend-adjusted. HAC standard errors & covariance, with lag=1, from AIC maximum lag=3, Bartlett kernel, and Newey-West fixed bandwidth=4.

Table (8)
Long-Run Covariance Matrix
Pre-whitening with lag=1. Max. Lags =3, Bartlett Kernel with Newey-West Bandwidth=4

	CO_2	Global Temp	Northern H Temp	Southern H Temp
CO_2	5.56E-06	9.33E-05	0.000231	5.51E-05
Global Temp	9.33E-05	0.036636	0.033947	0.010704
Northern H	0.000231	0.033947	0.082385	-0.003650
Southern H	5.51E-05	0.010704	-0.003650	0.011986

Using other methods to estimate the kernel does not seem to alter the results.

Table (9a)

The Engle-Granger Tests for Cointegration between Global Temperature and Methane

OLS regressions

$$Temp_t = \alpha + \beta CH_4 + \psi_t$$

$Global\ Temperature\ (NOAA)$

Sample 1984 - 2019

Variable	Coefficient	Std. Error	t-Statistic	P value
α	-5.05	0.92	-5.45	0.0000
β	0.003	0.00053	5.98	0.0000

R-squared	0.55	Mean dependent var.	0.54
Adjusted R-squared	0.54	S.D. dependent var.	0.23
S.E. of regression	0.16	Akaike info criterion	-0.78
Sum squared res.	0.86	Schwarz criterion	-0.69
Log likelihood	16.04	Hannan-Quinn criterion	-0.75
F-statistic	42.07	Durbin-Watson stat	1.93
Prob. (F-statistic)	0.0000	Wald F-statistic	35.7
Prob. (Wald F-statistic)	0.000001		

HAC standard errors & covariance (Prewhitening with lags = 0 from AIC, max lag=3, Bartlett Kernel, Newey-West fixed bandwidth=4

Table (9b)

Test the Residuals for Unit Root – ADF – Engle-Granger

$$\Delta\psi_t = \theta + \rho\psi_{t-1} + \sum_{i=1}^{n} \lambda_i \Delta\psi_{t-i} + \upsilon_t$$

	t-Statistic	P value
AIC, lag=0	-5.69	0.0000
Modified AIC, lag=2	-2.74	0.0785

All other Information Criteria give similar results

Table (9c)

Error Correction

$$\Delta Temp_t = \delta\, \Delta \ln CH_4 + \pi\, \psi_{t-1} + u_t$$

Variable	Coefficient	Std. Error	t-Statistic	P value
δ	1.82	4.76	0.38	0.7051
π	-0.02	0.20	-0.09	0.9288

R-squared	-0.005	Mean dependent variable	-0.014
Adjusted R-squared	-0.035	S.D. dependent variable	0.158
S.E. of regression	0.1613	Akaike info criterion	-0.754
Sum squared res.	0.8594	Schwarz criterion	-0.665
Log likelihood	15.207	Hannan-Quinn criterion	-0.724
Durbin-Watson stat	1.91		

$\Delta Temp$ is trend-adjusted . Constant term not reported. HAC standard errors & covariance (Pre whitening with lags = 0 from AIC. Max lags = 3, Bartlett kernel, Newey-West fixed bandwidth =4.

Table (10a)
The Engle-Granger Tests for Cointegration between
Global Temperature and Sulfur Hexafluoride
OLS regressions

$$Temp_t = \alpha + \beta SF_6 + \psi_t$$

$Global\ Temperature\ (NOAA)$

Sample 2001- 2019

Variable	Coefficient	Std. Error	t-Statistic	P value
α	0.20	0.16	1.30	0.2087
β	0.07	0.20	2.94	0.0095

R-squared	0.22	Mean dependent variable	0.68
Adjusted R-squared	0.22	S.D. dependent variable	0.21
S.E. of regression	0.18	Akaike info criterion	-0.46
Sum squared res.	0.62	Schwarz criterion	-0.419
Log likelihood	5.46	Hannan-Quinn criterion	-0.461
Durbin-Watson stat	1.76		

HAC standard errors and covariance (pre whitening, lag=0, from AIC with maximum lag=2, Bartlett kernel, Newey-West with fixed bandwidth=3

Table (10b)
Test the Residuals for Unit Root – ADF – Engle-Granger

$$\Delta\psi_t = \theta + \rho\psi_{t-1} + \sum_{i=1}^{n} \lambda_i \Delta\psi_{t-i} + \upsilon_t$$

	t-Statistic	Prob.*
AIC, lag=0	-3.78	0.0116
Modified AIC, lag=2	-1.80	0.3601

MacKinnon (1996) critical values are computed from sample of 20 observations, thus maybe inaccurate for this smaller sample. Test critical value at the 5% level is -3.040391

Table (10c)

$$\Delta Temp_t = \delta\, \Delta \ln SF_6 + \pi\, \psi_{t-1} + u_t$$

	Coefficient	Std. Error	t-Statistic	Prob.
δ	-0.49	12.7	-0.04	0.9694
π	0.06	0.307	0.19	0.8443

R-squared	0.004	Mean dependent variable	0.006
Adjusted R-squared	0.13	S.D. dependent variable	0.188
S.E. of regression	0.20	Akaike info criterion	-0.223
Sum squared res.	0.60	Schwarz criterion	-0.074
Log likelihood	5.0	Hannan-Quinn criterion	-0.202
F-statistic	0.03	Durbin-Watson stat	1.866
Prob.(F-statistic)	0.97	Wald F-statistic	0.025
Prob.(Wald F-statistic)	0.97		

HAC standard errors and covariance (pre whitening, lag=0, from AIC with maximum Lag=2, Bartlett kernel, Newey-West with fixed bandwidth=3.

Table (11)
Johansen's ML Test Results for "no" Cointegration
Sample 2001-2019

Trend assumption: Linear deterministic trend (restricted)
Series: GTEMP CO2 CH4 SF6
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	P value**
None *	0.942719	105.5181	63.87610	0.0000
At most 1 *	0.745019	51.18212	42.91525	0.0061
At most 2	0.611558	25.21733	25.87211	0.0601
At most 3	0.317244	7.250726	12.51798	0.3191

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	P value**
None *	0.942719	54.33602	32.11832	0.0000
At most 1 *	0.745019	25.96479	25.82321	0.0479
At most 2	0.611558	17.96660	19.38704	0.0794
At most 3	0.317244	7.250726	12.51798	0.3191

- Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
- * denotes rejection of the hypothesis at the 0.05 level
- **MacKinnon-Haug-Michelis (1999) p-values
- We don't report the results, but we also carried out the tests with different assumptions about trend

Table (12)
Greenhouse gases effect on Temperature

	OLS (i)						Dynamic OLS ii					
Dependent variable=>	Global Temp		Northern Hemp		Southern Hemp		Global Temp		Northern Hemp		Southern Hemp	
	a	b	c	d	e	f	g	h	i	j	k	l
CO2	-0.016	0.25	-0.005	0.14	-0.005	0.03	-0.05	0.18	0.02	0.31	0.00	0.03
	(0.3180)	(0.0000)	(0.6958)	(0.0003)	(0.1349)	(0.2127)	(0.2624)	(0.2536)	(0.6314)	(0.0057)	(0.9556)	(0.6259)
CH4	0.003	0.02	0.001	0.012	0.001	0.004	0.01	0.03	-0.004	0.01	-0.00	0.009
	(0.3020)	(0.0000)	(0.6754)	(0.0006)	(0.1018)	(0.0949)	(0.2502)	(0.0484)	(0.6501)	(0.0353)	(0.9811)	(0.0715)
SF6	0.137	-2.30	0.13	-1.12	0.07	-0.24	0.25	-1.89	0.15	-2.5	0.06	-0.43
	(0.0646)	(0.0000)	(0.0452)	(0.0007)	(0.0000)	(0.2783)	(0.1776)	(0.1726)	(0.3709)	(0.0094)	(0.0686)	(0.3982)
Cons	-	-124.9	-	-67.4	-	-16.25	-	-108.03	-	-132.2	-	-25.66
		(0.0000)		(0.0003)		(0.1685)		(0.1293)		(0.0071)		(0.3277)
$\bar{R}^2$	0.29	0.50	0.71	0.77	0.64	0.63	0.31	0.33	0.67	0.82	0.69	0.73
DW	1.94	2.4	1.36	1.72	1.78	1.89	-	-	-	-	-	-
σ	0.17	0.15	0.12	0.10	0.07	0.07	0.17	0.17	0.12	0.08	0.07	0.05
Σ	-	-	-	-	-	-	0.025	0.01	0.02	0.003	0.004	0.001

i-The sample is 1998 – 2019 (22 observations). The sample is 2000-2018 for the Dynamic OLS. **a**-HAC standard errors and covariance (pre-whitening with lags=0 AIC, max lag=2, Bartlett kernel, Newey-West bandwidth 2.8103, lag length=2. **b**-HAC standard errors and covariance (pre-whitening with lags=2 AIC, max lag=2, Bartlett kernel, Newey-West bandwidth 1.9982, lag length=2. **c**-HAC standard errors and covariance (pre-whitening with lags=2 AIC, max lag=2, Bartlett kernel, Newey-West bandwidth 2.2124, lag length=2. **d**-HAC standard errors and covariance (pre-whitening with lags=2 AIC, max lag=2, Bartlett kernel, Newey-West bandwidth 1.1849, lag length=2. **e**-HAC standard errors and covariance (pre-whitening with lags=2 AIC, max lag=2, Bartlett kernel, Newey-West bandwidth 1.2698, lag length=2. **f**- HAC standard errors and covariance (pre-whitening with lags=2 AIC, max lag=2, Bartlett kernel, Newey-West bandwidth 1.6634, lag length=2. **g, h, I, j, k, and l**- Fixed lag=lead=1, long-run variance estimate includes pre-whitening, Bartlett kernel, Newey-West bandwidth fixed=3. **ii**-fixed lags and leads=1. Pre-whitening with lag=1, Bartlett kernel, Newey-West fixed bandwidth = 3. P values are in parentheses. σ is the standard error of the regression and Σ is the long-run variance.

Table (13)
Variance Decomposition of Global Temperature

Period	S.E.	Δ *ln*SF6	Δ *ln*CH4	Δ *ln*CO2	Trend-adjusted Temperature
1	0.236795	0.030674	2.476814	58.28683	39.20568
		(7.39166)	(8.08993)	(14.1119)	(13.0130)
2	0.241270	2.131906	3.229579	56.81478	37.82374
		(12.1137)	(10.6268)	(14.4931)	(12.3727)
3	0.244424	2.383563	4.689667	55.43510	37.49167
		(14.0016)	(11.7295)	(14.2587)	(12.5337)
4	0.244800	2.442591	4.821598	55.26749	37.46832
		(14.4288)	(12.1398)	(14.1431)	(12.2748)
5	0.245527	2.637693	4.828121	55.24373	37.29046
		(14.7702)	(12.4129)	(14.2589)	(12.2768)
6	0.245980	2.816144	4.920739	55.10326	37.15986
		(15.5282)	(12.9027)	(14.4321)	(12.5087)
7	0.246031	2.823351	4.929198	55.09583	37.15162
		(16.3114)	(13.2672)	(14.5023)	(12.6993)
8	0.246091	2.853454	4.932172	55.07628	37.13809
		(17.1307)	(13.5717)	(14.7595)	(12.9287)
9	0.246241	2.934683	4.959746	55.01017	37.09540
		(17.6784)	(13.9406)	(14.9645)	(13.0507)
10	0.246331	2.973532	4.971177	54.97996	37.07533
		(18.2617)	(14.2735)	(15.1808)	(13.3016)

Table (14)
Variance Decomposition of Global Temperature

Period	S.E.	Fossil fuel consumption growth rate	Δ lnSF6	Δ lnCH4	Δ lnCO2	Trend-adjusted Temperature
1	0.187967	0.053653	1.732646	15.14369	15.34214	67.72787
		(8.32800)	(10.5581)	(13.1705)	(11.3110)	(15.4309)
2	0.241854	30.53246	10.02299	9.251383	9.277542	40.91563
		(17.9747)	(13.3192)	(11.1544)	(7.80495)	(12.9214)
3	0.248600	31.24406	9.493994	8.759872	9.000673	41.50141
		(16.3244)	(11.7002)	(12.4046)	(6.87794)	(11.3502)
4	0.252693	30.24314	9.415166	11.40362	8.767874	40.17020
		(15.0044)	(10.3668)	(13.5127)	(6.22119)	(10.7994)
5	0.259701	32.07307	9.135137	10.98669	8.430248	39.37485
		(15.7756)	(10.7418)	(13.4952)	(6.15082)	(11.5816)
6	0.263012	31.52398	9.697861	10.95134	8.245082	39.58174
		(15.5607)	(11.9583)	(13.1147)	(5.86024)	(11.0049)
7	0.265332	31.80558	9.739780	10.86688	8.124656	39.46311
		(14.9419)	(13.0516)	(14.0147)	(5.63982)	(11.0574)
8	0.265868	31.70846	9.720572	11.02949	8.141462	39.40001
		(15.0379)	(14.2863)	(14.1229)	(5.79997)	(10.9845)
9	0.266695	31.82086	9.672300	11.13047	8.158770	39.21760
		(15.7584)	(15.9432)	(13.9012)	(5.88292)	(11.5821)
10	0.267697	31.61276	9.907188	11.17639	8.127355	39.17631
		(15.4627)	(17.0779)	(14.7270)	(5.84513)	(11.8342)

Table (15)
Variance decomposition of Northern hemisphere temperature

Period	S.E.	Fossil fuel consumption growth rate	$\Delta\, ln$SF6	$\Delta\, ln$CH4	$\Delta\, ln$CO2	Trend-adjusted Temperature
1	0.117170	8.016926	3.712420	17.07571	6.034037	65.16090
		(13.2481)	(9.76967)	(13.5215)	(8.34940)	(16.5440)
2	0.162640	19.71743	15.52897	9.056645	3.622128	52.07483
		(17.9387)	(14.5820)	(10.4056)	(6.15033)	(15.9073)
3	0.164544	19.98890	15.24164	9.720973	3.696593	51.35189
		(17.8770)	(13.4788)	(12.4004)	(5.39707)	(14.9991)
4	0.168619	19.22421	16.26656	10.01850	3.525038	50.96569
		(17.4944)	(14.0730)	(12.4299)	(4.88194)	(14.3568)
5	0.179030	17.61522	18.60861	13.15459	3.324177	47.29740
		(17.8327)	(14.7541)	(12.9154)	(4.64329)	(14.1406)
6	0.182755	17.16337	20.07363	13.27285	3.264060	46.22609
		(18.0378)	(15.6088)	(13.0292)	(4.50480)	(13.8289)
7	0.185000	17.75507	20.92907	12.96373	3.185629	45.16650
		(18.2754)	(16.3890)	(13.0835)	(4.44336)	(13.8368)
8	0.186155	17.53605	21.83087	12.86499	3.148727	44.61936
		(19.0722)	(17.3864)	(13.3739)	(4.34207)	(13.9248)
9	0.187454	17.53440	22.60061	12.75111	3.107100	44.00678
		(19.2969)	(18.0005)	(13.5342)	(4.31492)	(13.9582)
10	0.188980	17.27415	23.51253	12.76654	3.058255	43.38853
		(19.6587)	(18.6416)	(14.0953)	(4.27284)	(14.0795)

Table (16)
Variance decomposition of the Southern hemisphere temperature

Period	S.E.	Fossil fuel consumption growth rate	Δ *ln*SF6	Δ *ln*CH4	Δ *ln*CO2	Trend-adjusted Temperature
1	0.070704	56.17321	5.802424	5.558556	18.36526	14.10054
		(15.4827)	(7.64937)	(6.82489)	(9.16113)	(5.78022)
2	0.081120	57.04799	8.602236	8.801034	14.71535	10.83339
		(15.9829)	(10.3430)	(10.6381)	(7.97124)	(4.81782)
3	0.083330	54.45555	8.957269	11.92328	13.96009	10.70380
		(15.8577)	(11.2199)	(12.5427)	(7.59763)	(5.13591)
4	0.085976	53.61120	8.526886	14.68414	13.11468	10.06308
		(16.1920)	(11.6180)	(12.9440)	(7.29741)	(4.90116)
5	0.088855	51.86729	12.35492	14.02817	12.27864	9.470983
		(16.8286)	(12.5986)	(12.8827)	(7.20221)	(4.90914)
6	0.092117	50.12100	15.46873	13.13571	11.88057	9.393991
		(17.3522)	(13.8748)	(13.4098)	(7.50797)	(5.16762)
7	0.092873	50.03039	15.82761	13.12547	11.73282	9.283701
		(17.8113)	(14.4445)	(13.9344)	(7.81177)	(5.29569)
8	0.093064	49.91271	16.04199	13.10869	11.68527	9.251342
		(18.1647)	(15.0926)	(14.3663)	(7.86838)	(5.36737)
9	0.093655	49.36446	16.94259	13.00322	11.53894	9.150793
		(18.6952)	(15.8866)	(14.5478)	(8.04022)	(5.64798)
10	0.094252	48.78942	17.55280	13.10969	11.46177	9.086320
		(19.1289)	(16.5267)	(15.1030)	(8.19411)	(5.65854)

Figure (1)

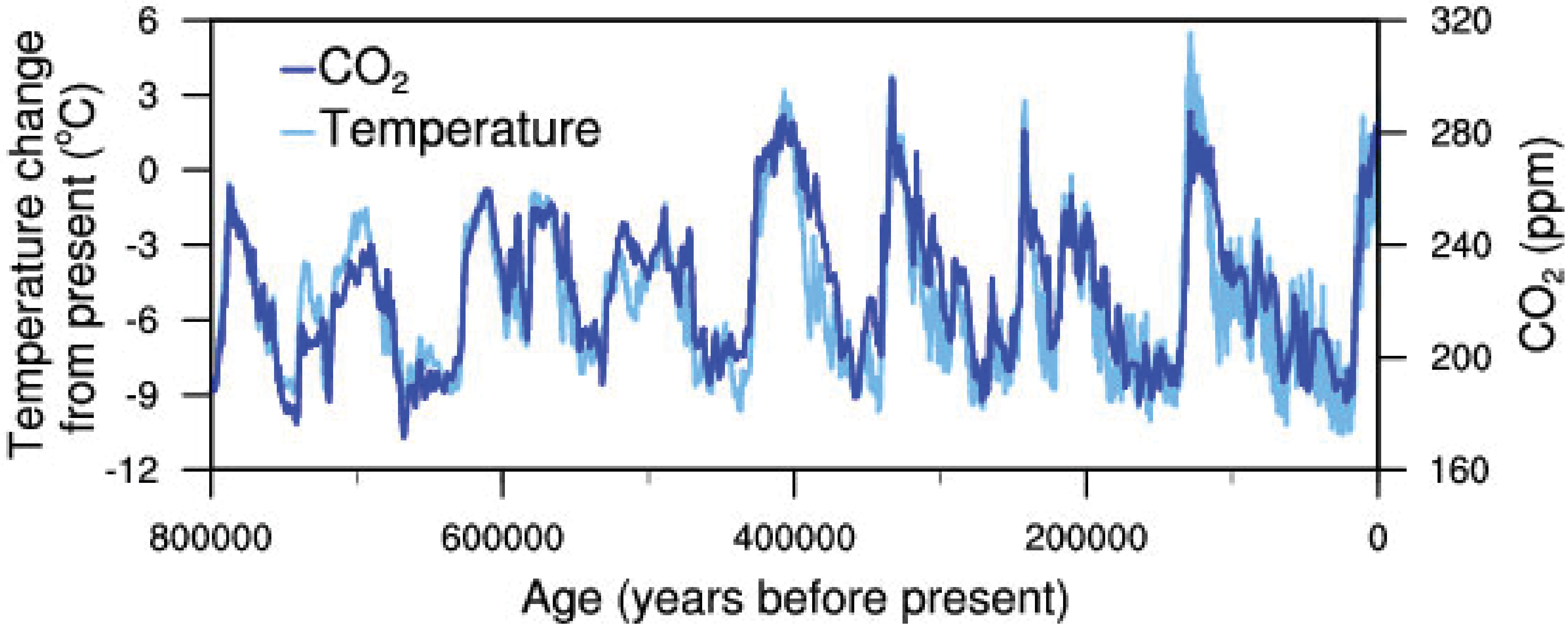

Temperature change from present (°C)
6
3
0
-3
-6
-9
-12
CO2
Temperature
320
280
240
200
160
CO2 (ppm)
800000
600000
400000
200000
0
Age (years before present)

Figure (2)
Trend is visually clear

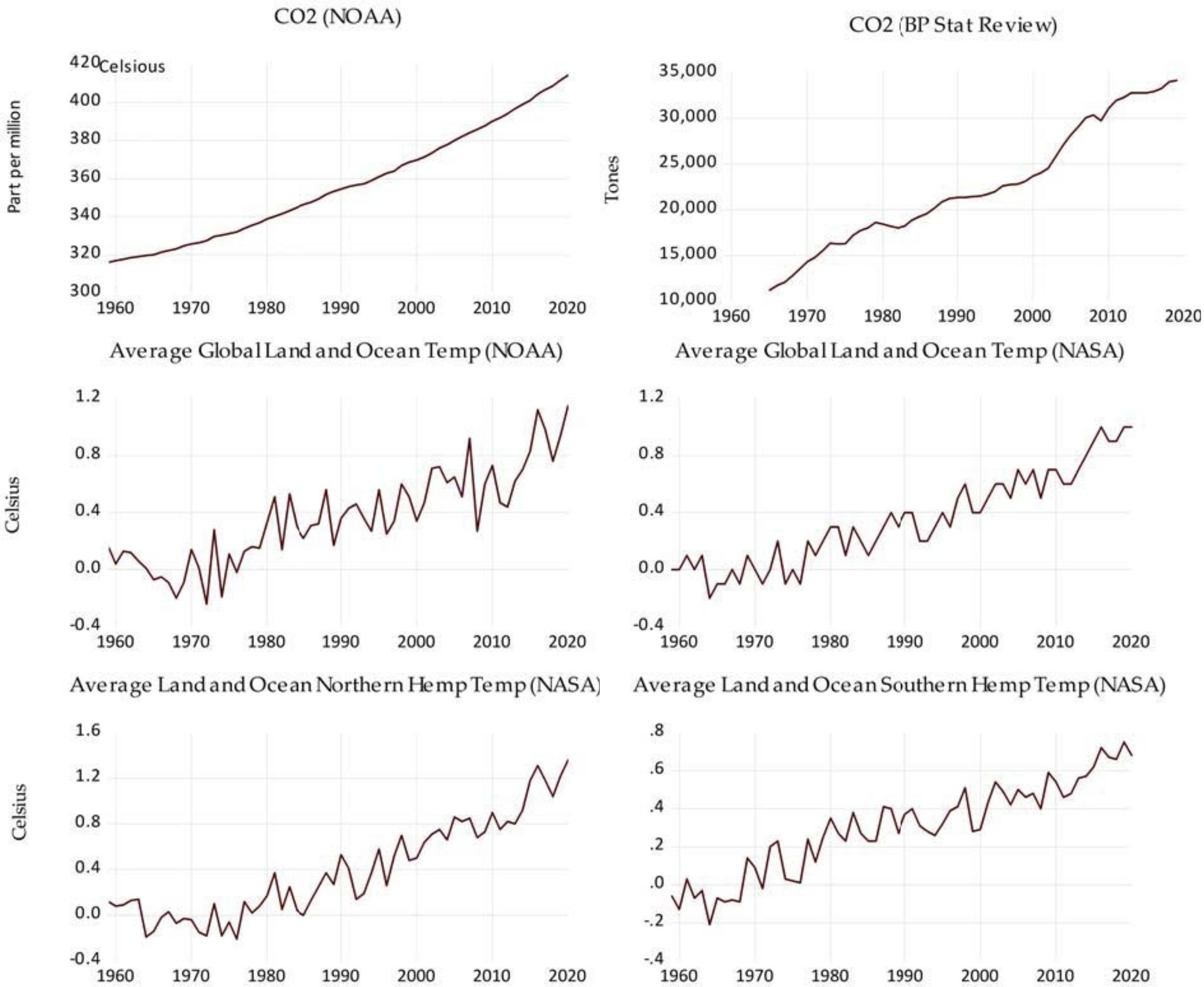


Figure (3)
Spurious Correlation

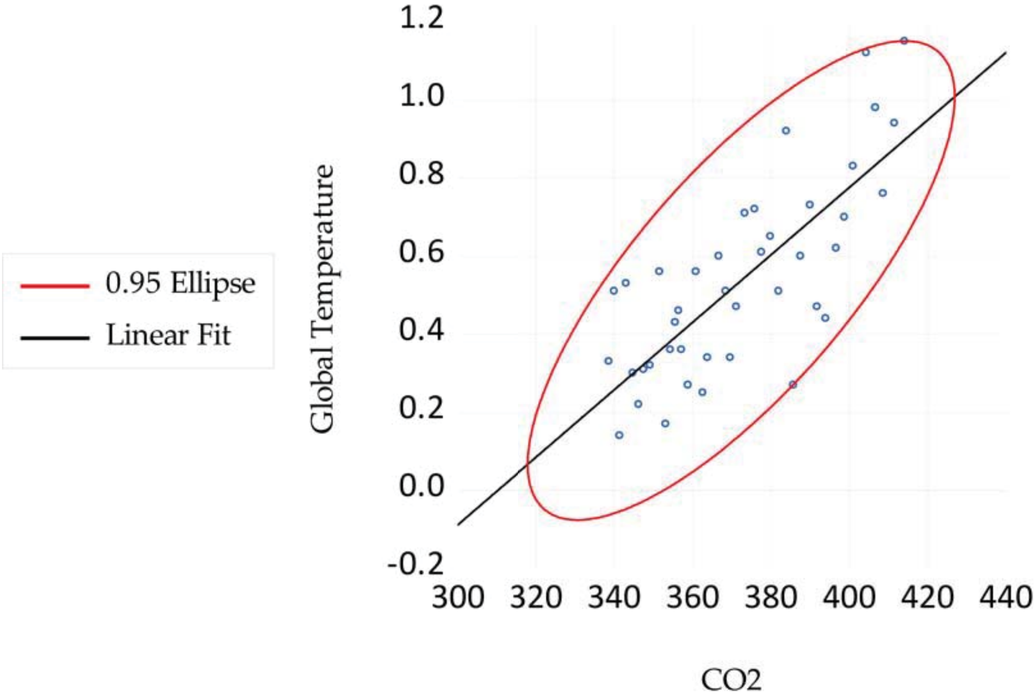


Figure (4a)

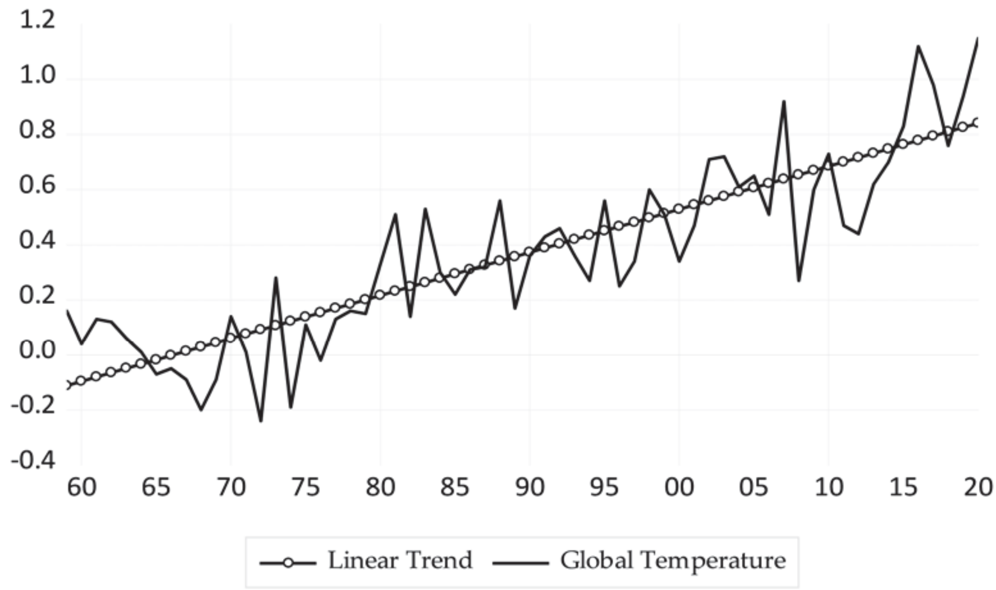


Figure (4b)

Trend-Adjusted Global Temperature

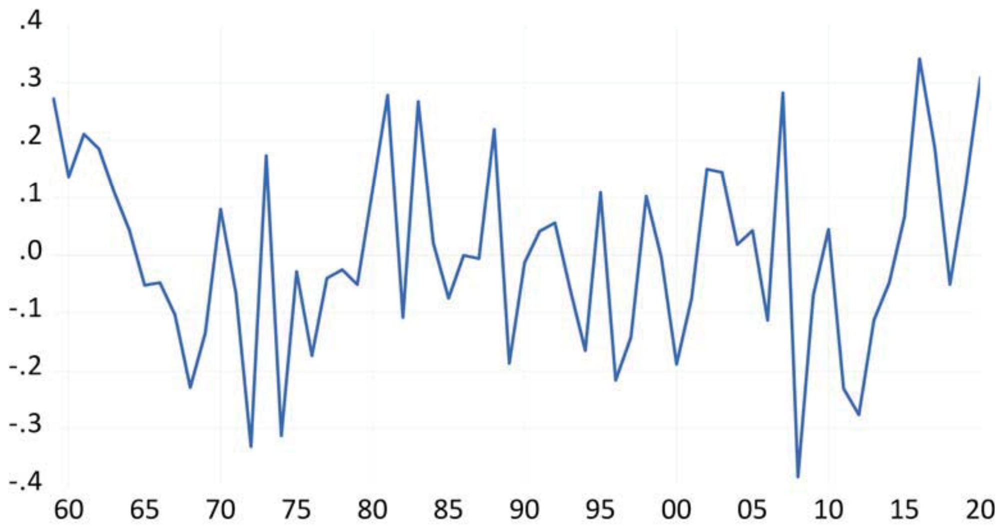


Figure (4c)

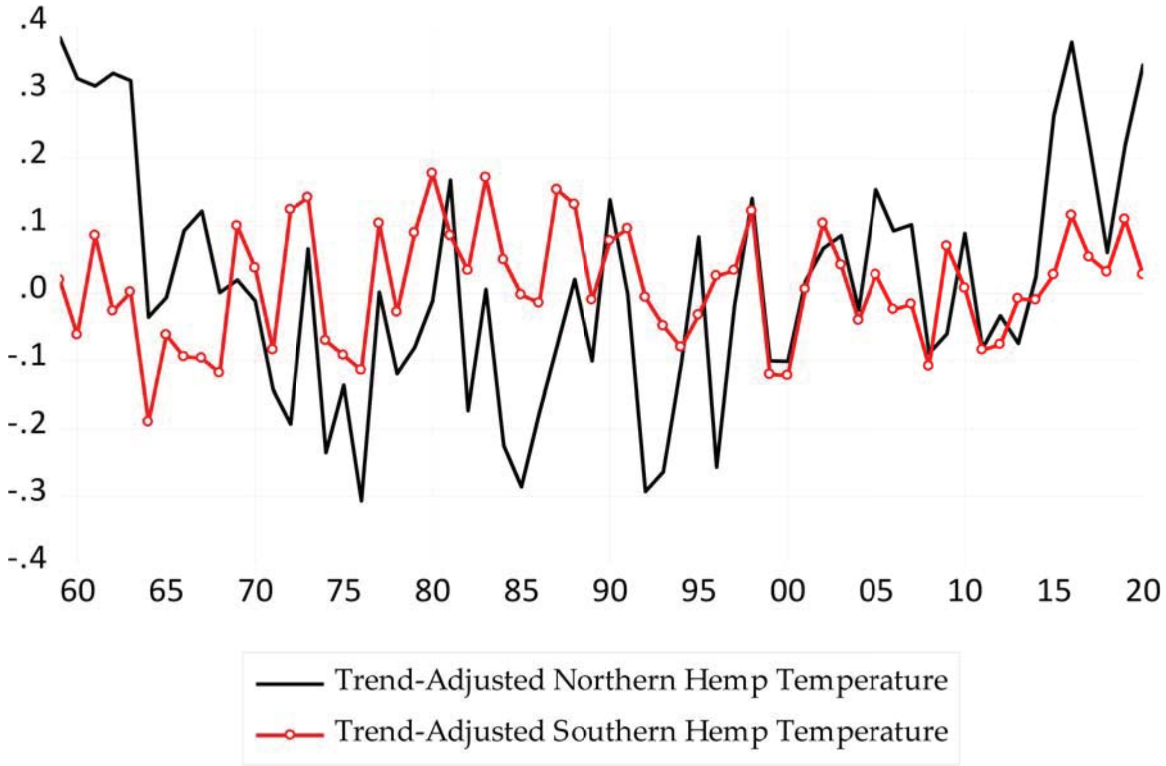

.4
.3
.2
.1
.0
-.1
-.2
-.3
-.4
60
65
70
75
80
85
90
95
00
05
10
15
20
Trend-Adjusted Northern Hemp Temperature
Trend-Adjusted Southern Hemp Temperature

Figure (5a)

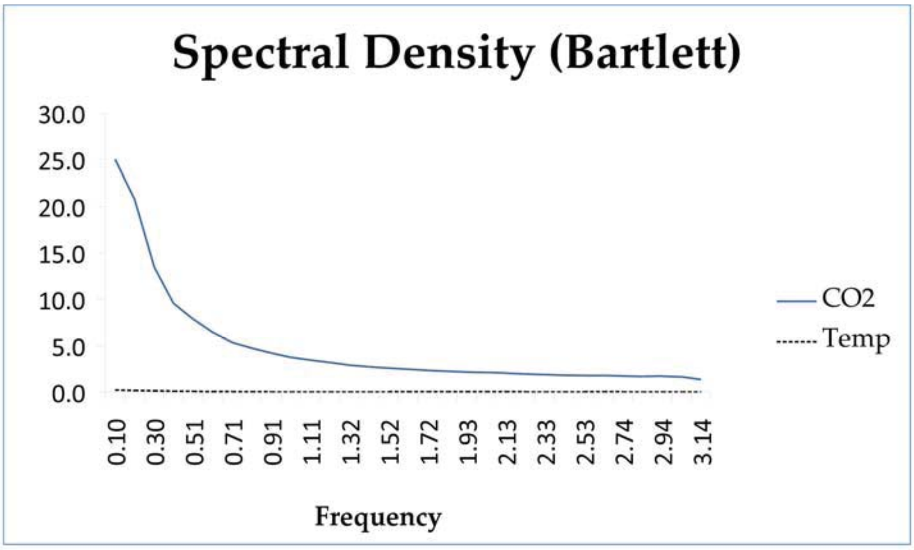


Figure (5b)

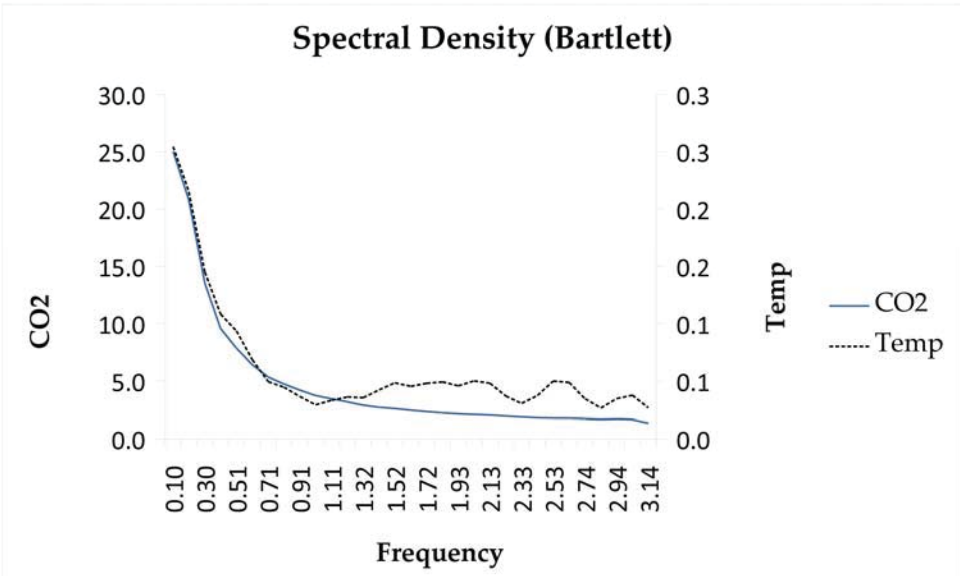
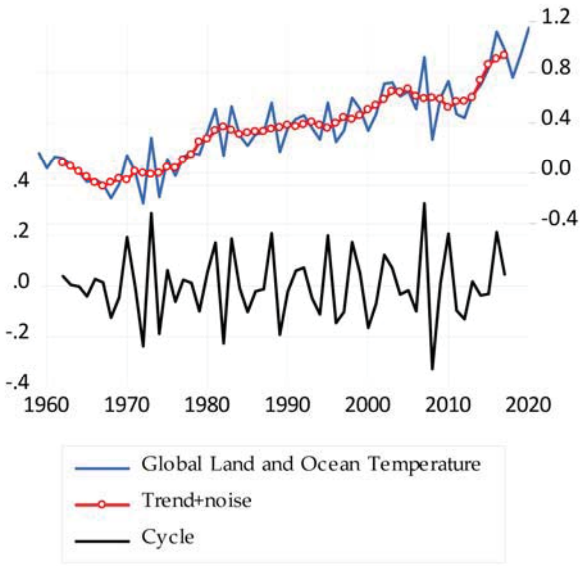


Figure (6)

Fixed Length Symmetric (Christiano-Fitzgerald) Filter



Frequency Response Function

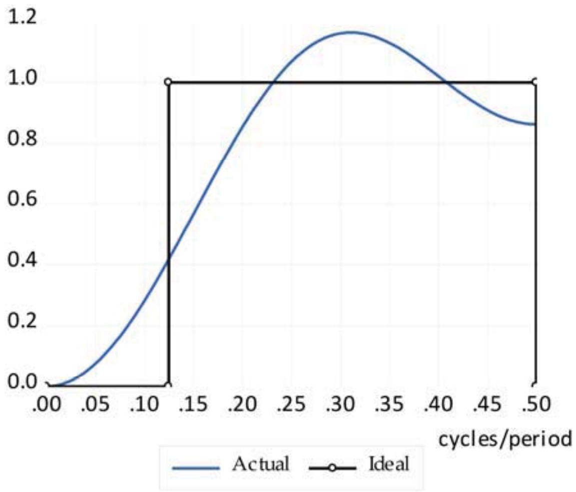
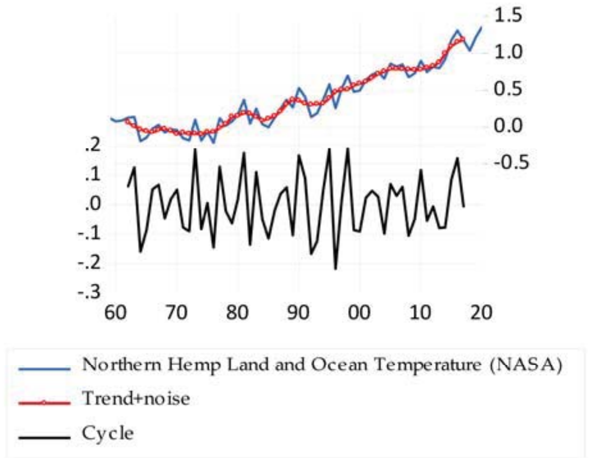


Figure (7)

Fixed Length Symmetric (Christiano-Fitzgerald) Filter



Frequency Response Function

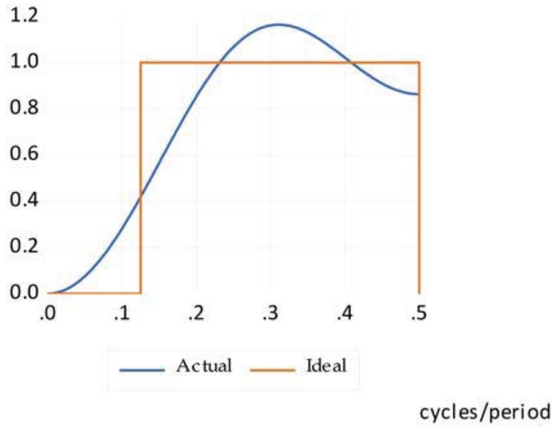
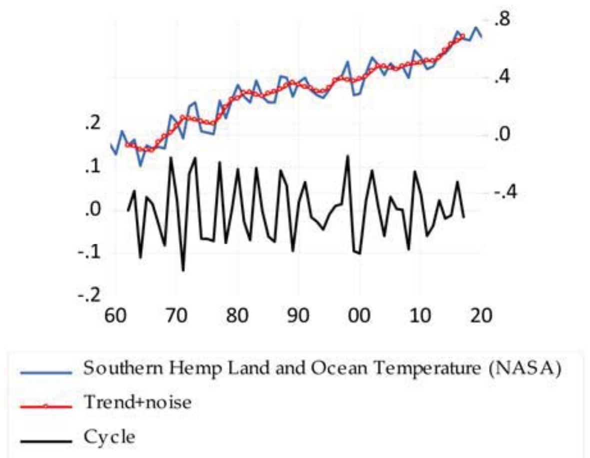


Figure (8)

Fixed Length Symmetric (Christiano-Fitzgerald) Filter



Frequency Response Function

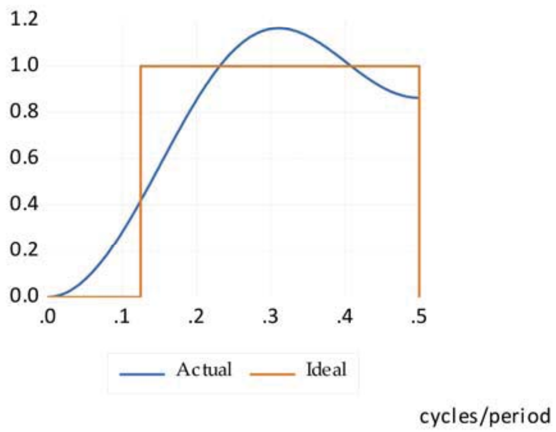


Figure (9)

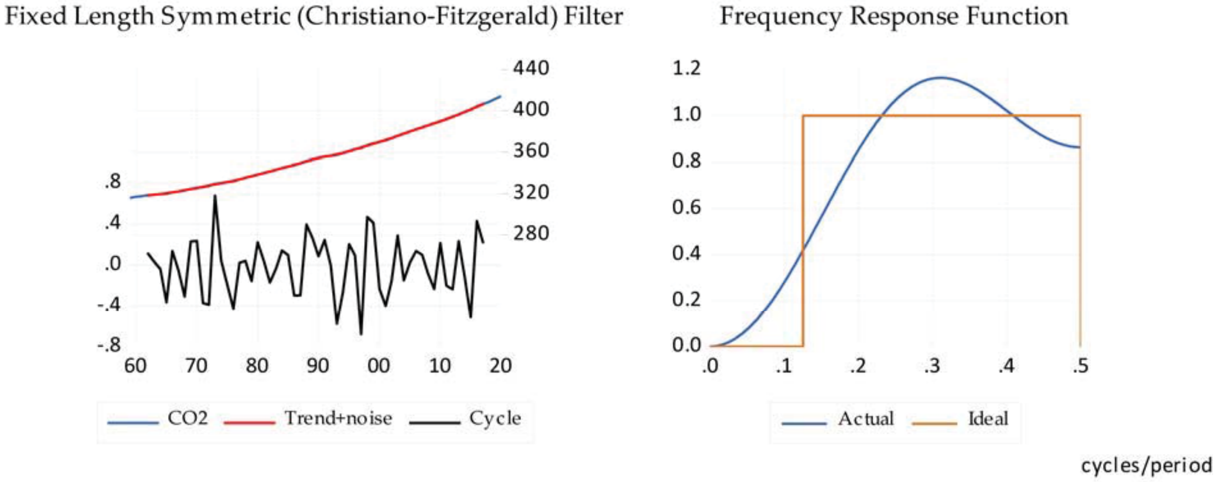


Figure (10)

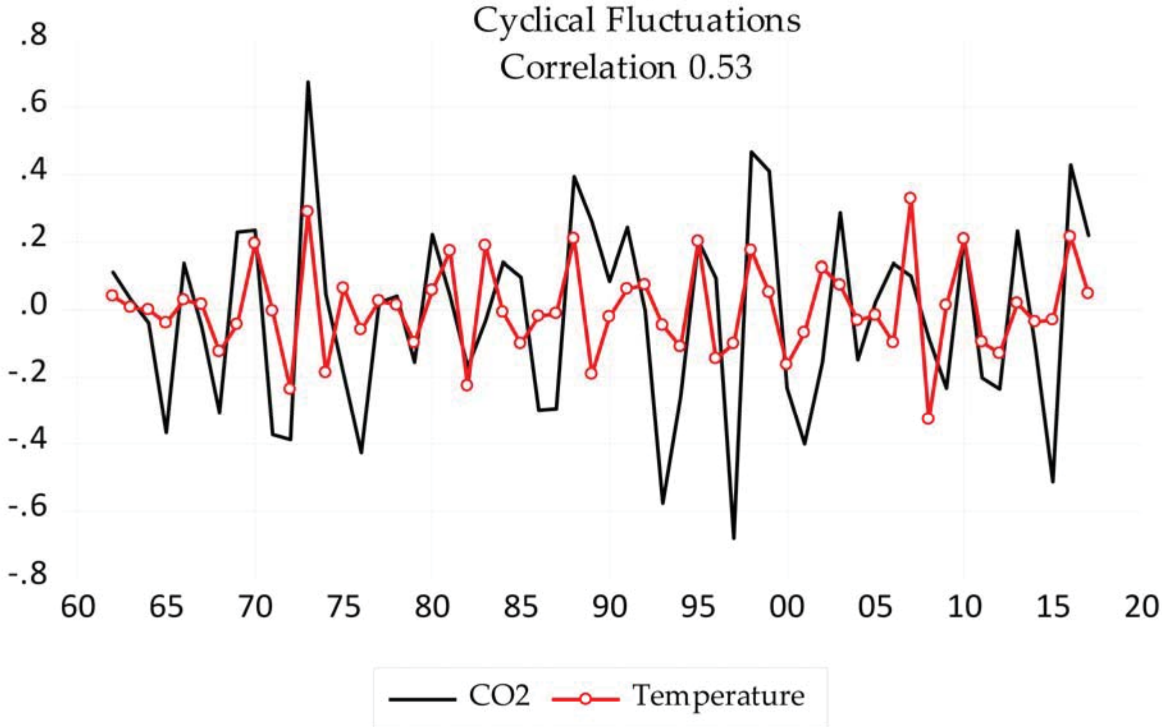


Figure (11a)

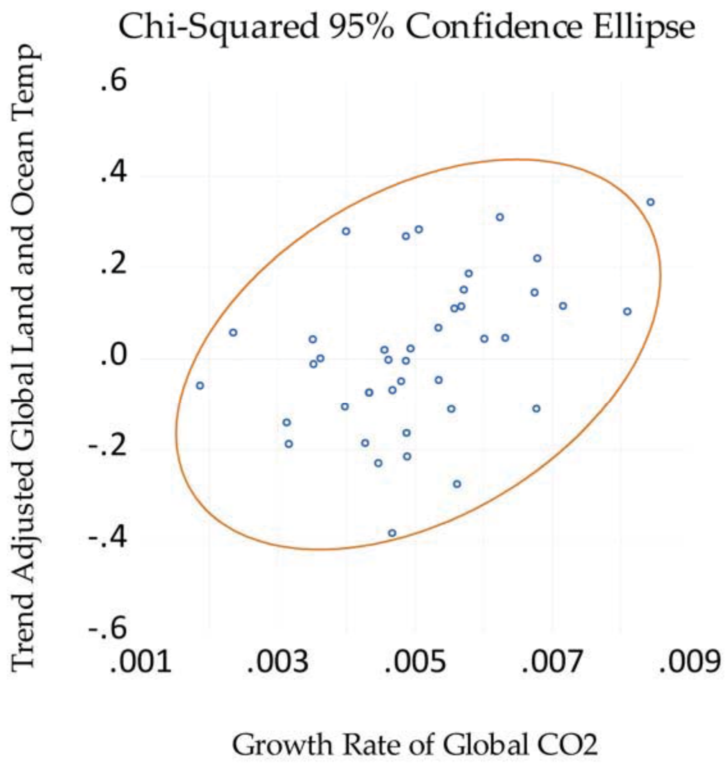


Figure (12a)

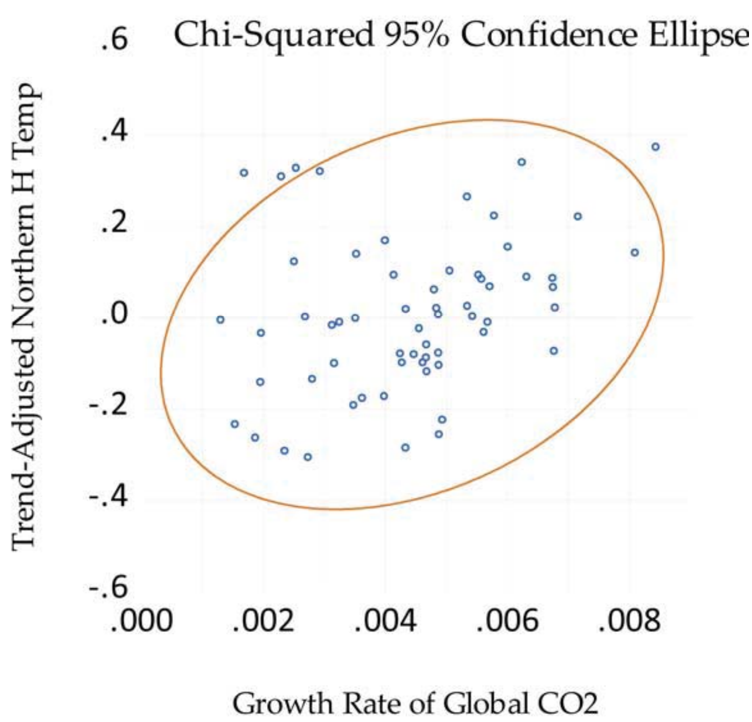


Figure (13a)

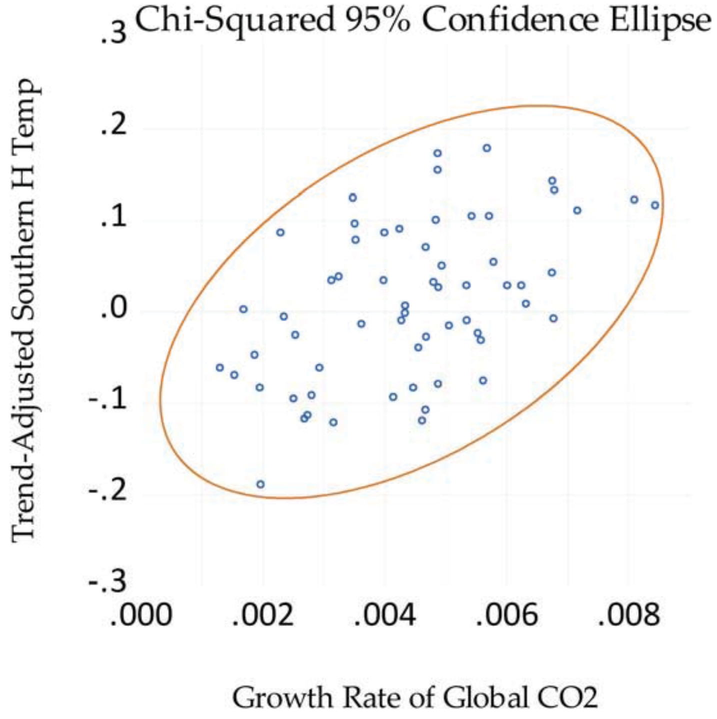


Figure (11b)

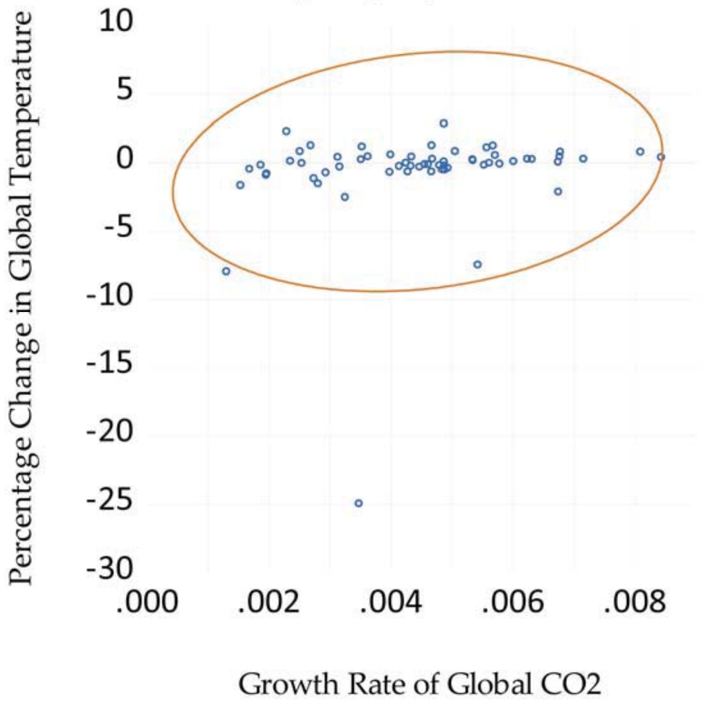


Figure (12b)

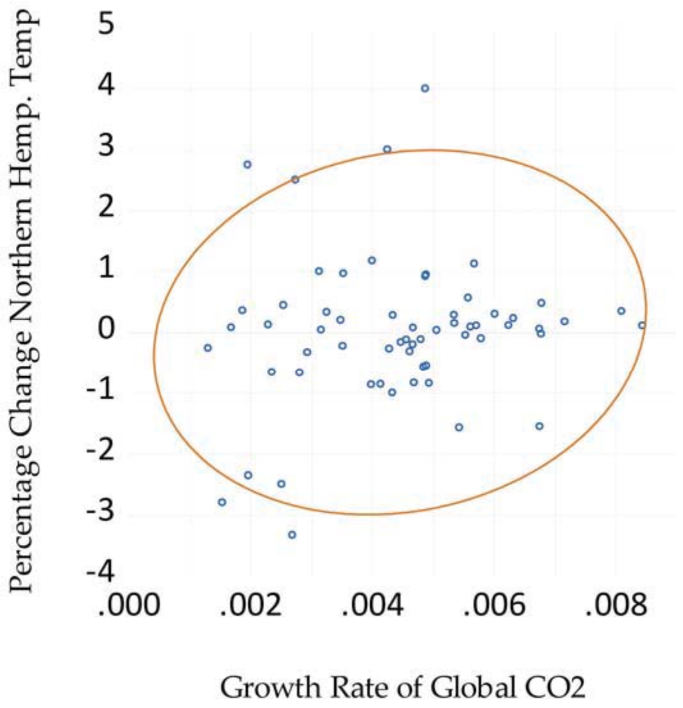


Figure (13b)

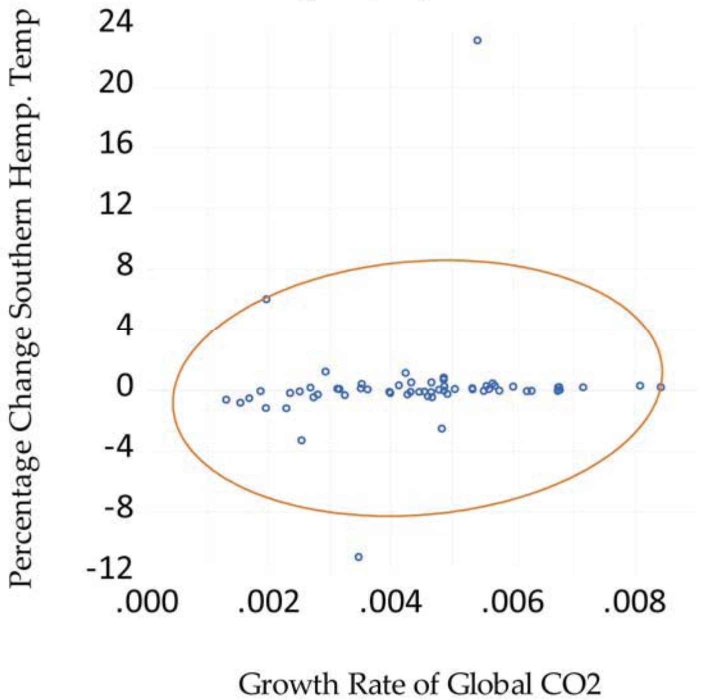


Figure (14)
D denotes log differenced
Variance Decomposition using Cholesky (d.f. adjusted) Factors ± 2 S.E.

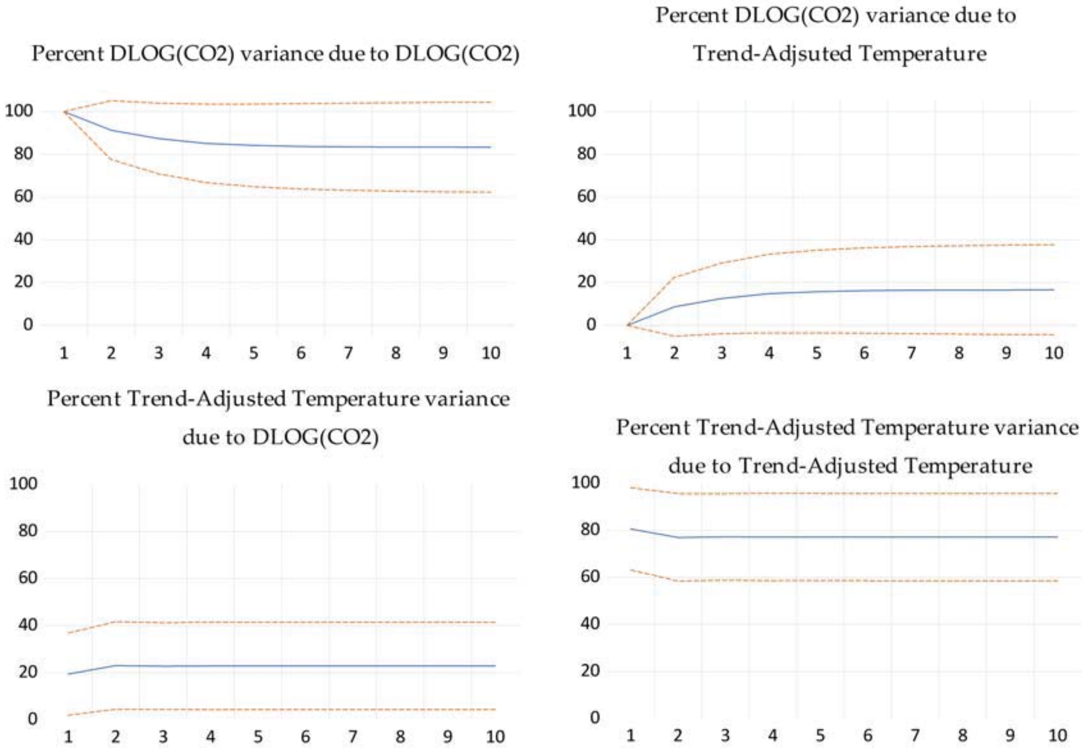


Figure (15)
D denotes the log-differenced
Variance Decomposition using Cholesky (d.f. adjusted) Factors ± 2 S.E.

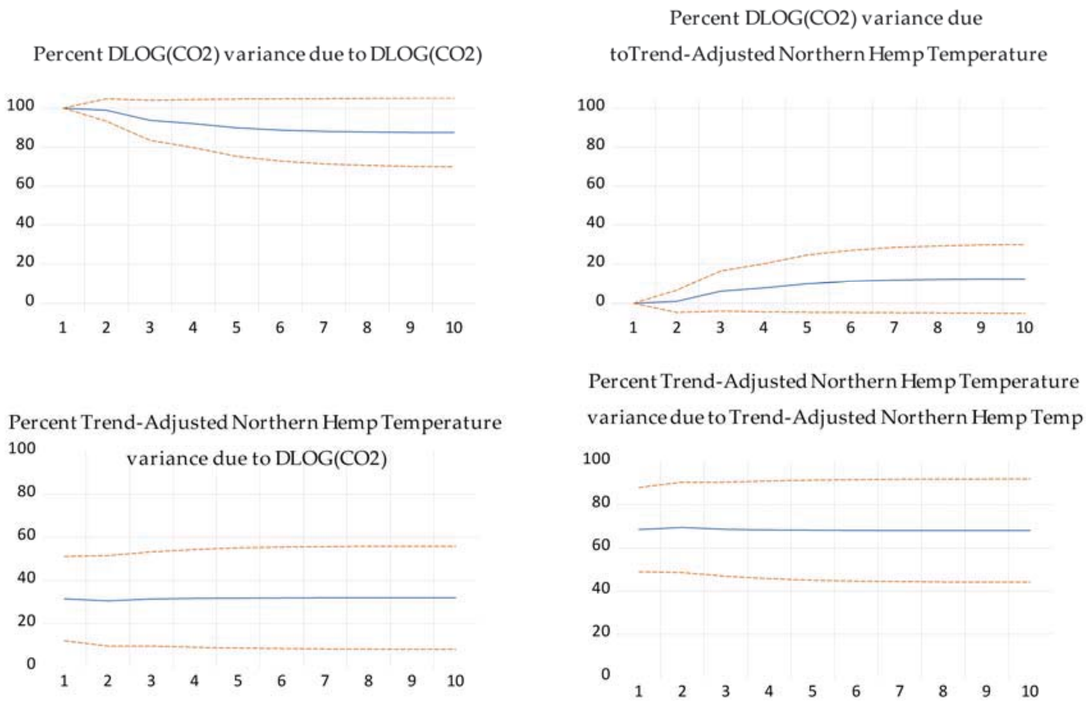


Figure (16)
D denotes log-differenced

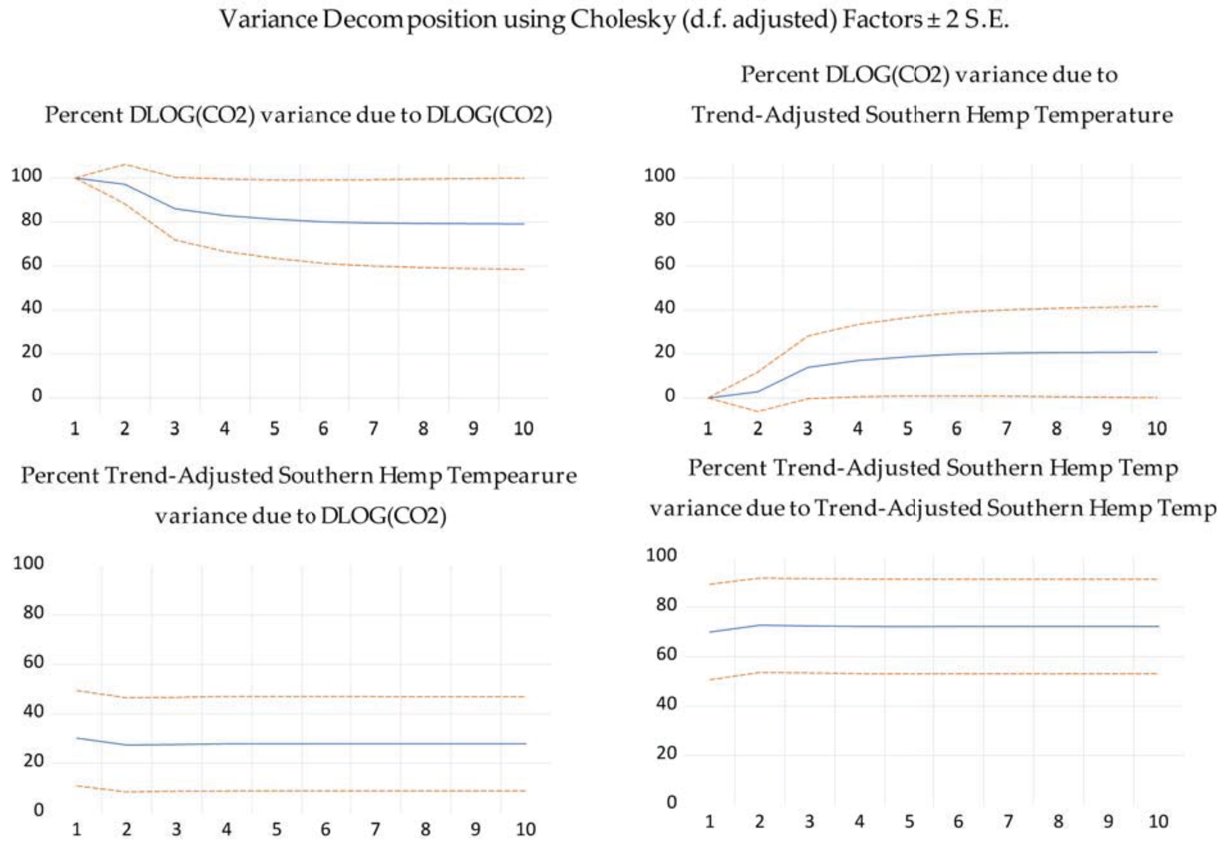


Figure (17)

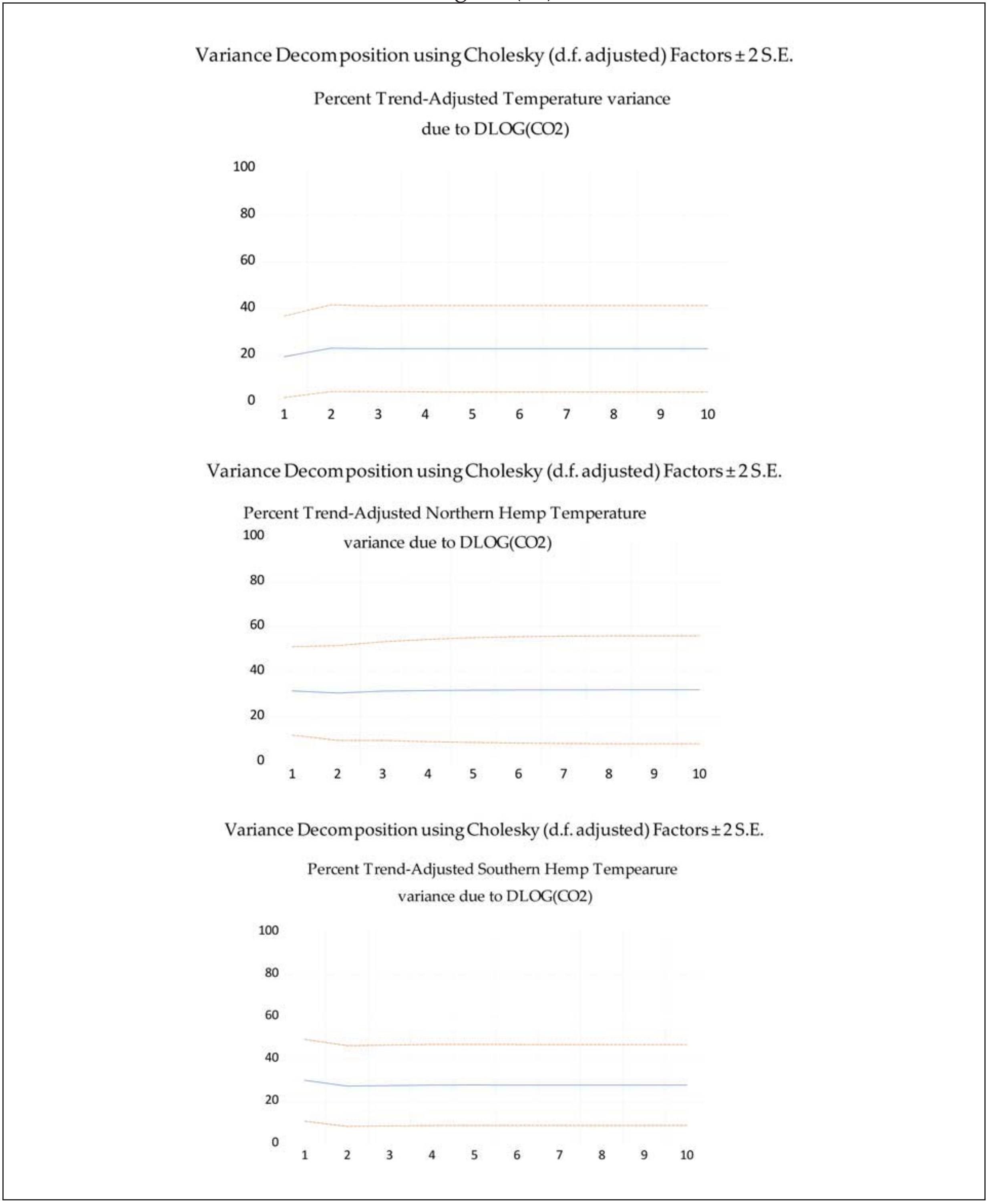
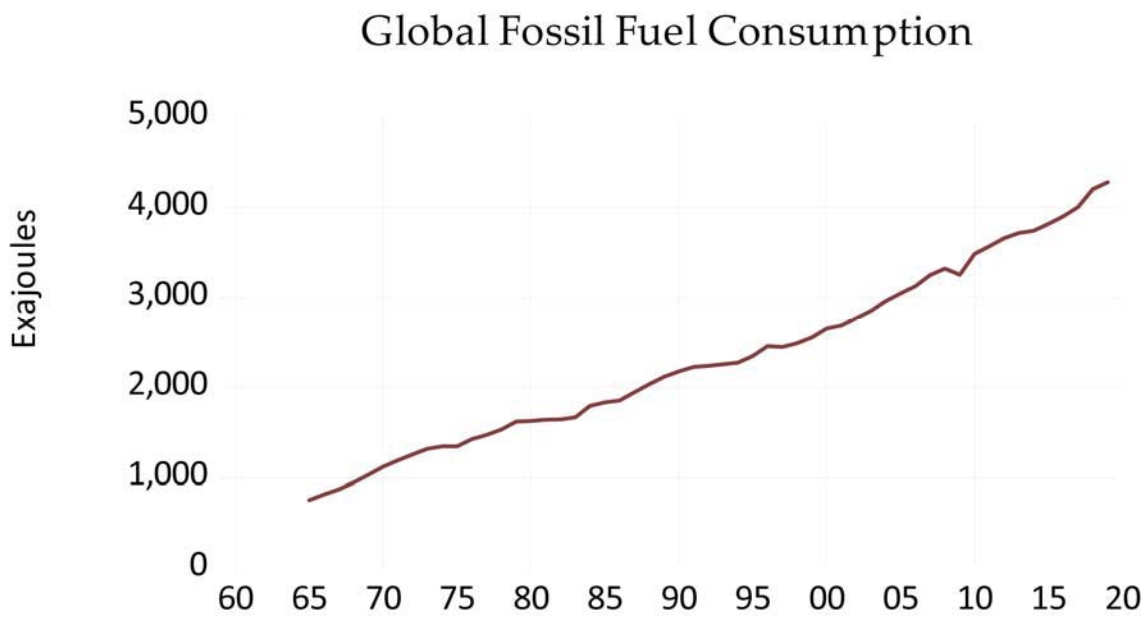

Variance Decomposition using Cholesky (d.f. adjusted) Factors ± 2 S.E.
Percent Trend-Adjusted Temperature variance due to DLOG(CO2)
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10
Variance Decomposition using Cholesky (d.f. adjusted) Factors ± 2 S.E.
Percent Trend-Adjusted Northern Hemp Temperature variance due to DLOG(CO2)
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10
Variance Decomposition using Cholesky (d.f. adjusted) Factors ± 2 S.E.
Percent Trend-Adjusted Southern Hemp Tempearure variance due to DLOG(CO2)
100
80
60
40
20
0
1 2 3 4 5 6 7 8 9 10

Figure (18)


Global Fossil Fuel Consumption
Exajoules
5,000
4,000
3,000
2,000
1,000
0
60 65 70 75 80 85 90 95 00 05 10 15 20

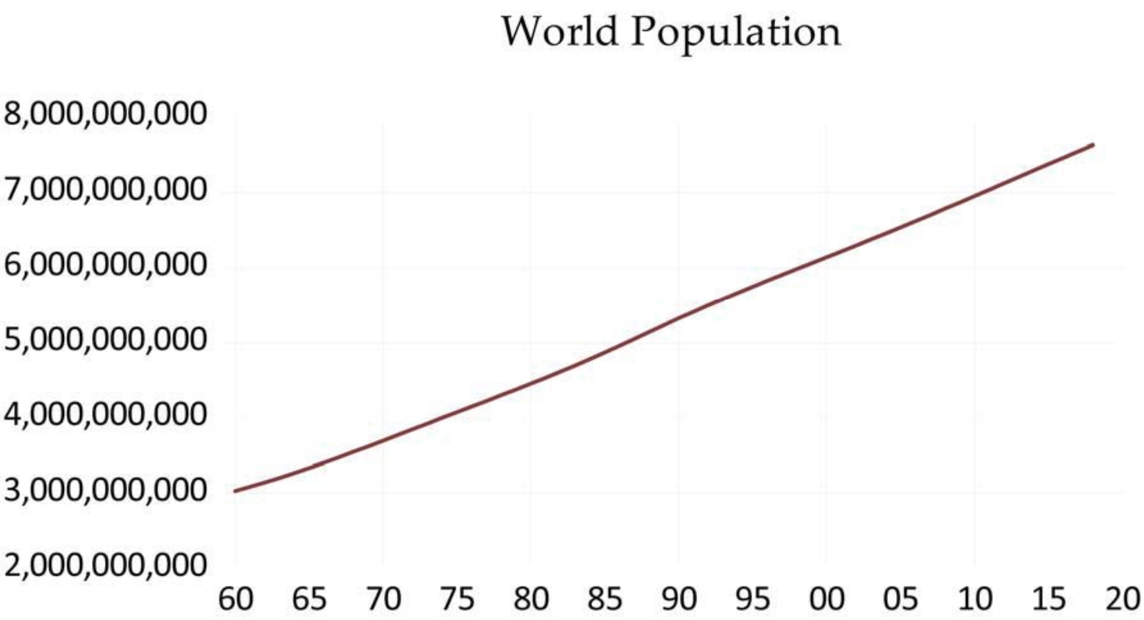

World Population
8,000,000,000
7,000,000,000
6,000,000,000
5,000,000,000
4,000,000,000
3,000,000,000
2,000,000,000
60 65 70 75 80 85 90 95 00 05 10 15 20

Figure (19a)

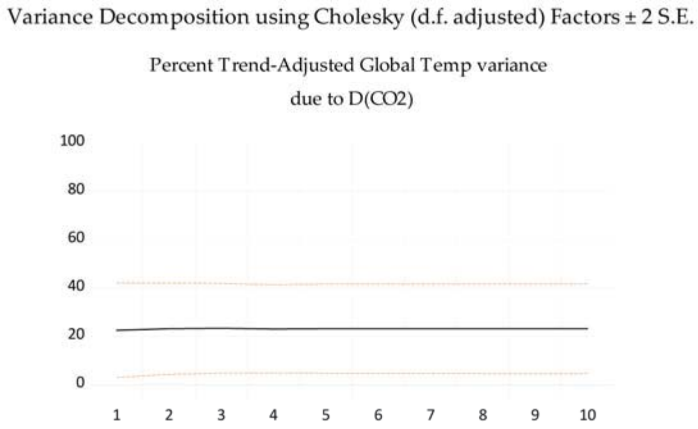


Figure (19b)

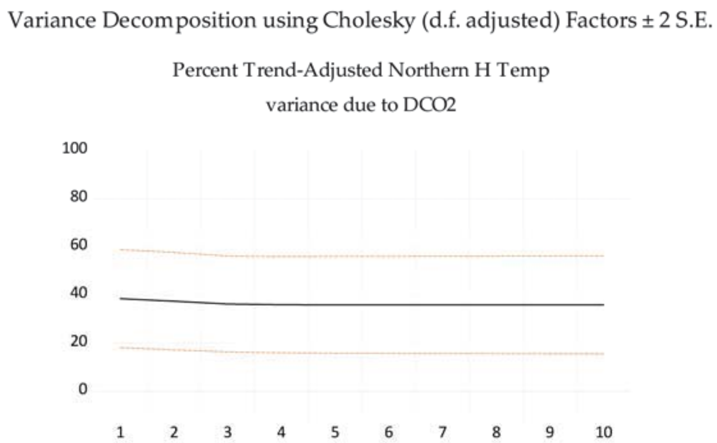


Figure (19c)

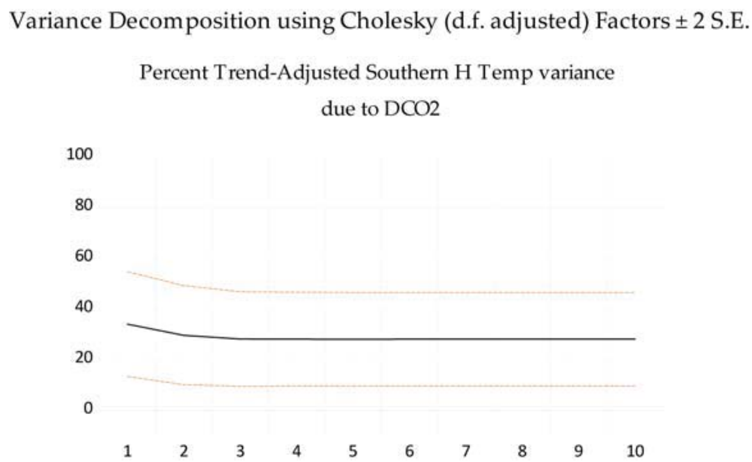


Figure (20a)

Variance Decomposition using Cholesky (d.f. adjusted) Factors ± 2 S.E.

Percent Trend-Adjusted Global Temperature variance due to DFOSSILFUEL

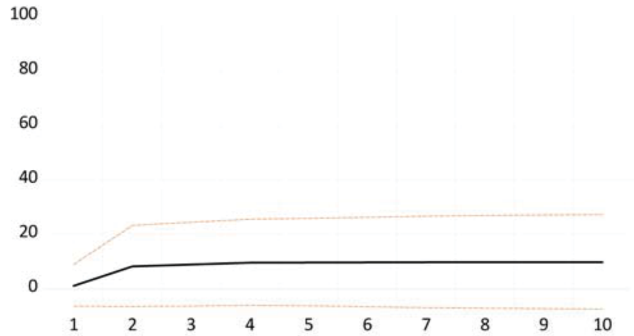


Figure (20b)

Variance Decomposition using Cholesky (d.f. adjusted) Factors ± 2 S.E.

Percent Trend-Adjusted Northern H Temo
variance due to D(Fossil Fuel Consumption)

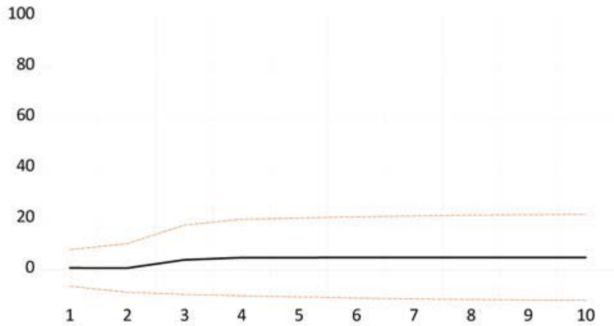


Figure (20c)

Variance Decomposition using Cholesky (d.f. adjusted) Factors ± 2 S.E.

Percent Trend-Adjusted Southern H Temp Variance
Due to D(Fossil Fuel Consumption)

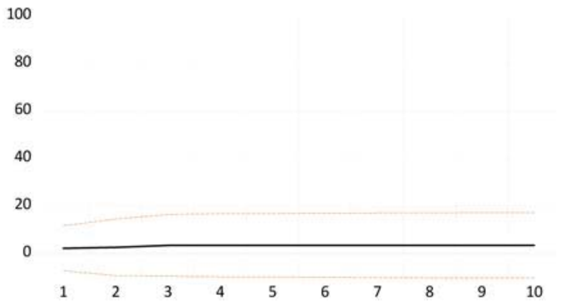


Figure (21)

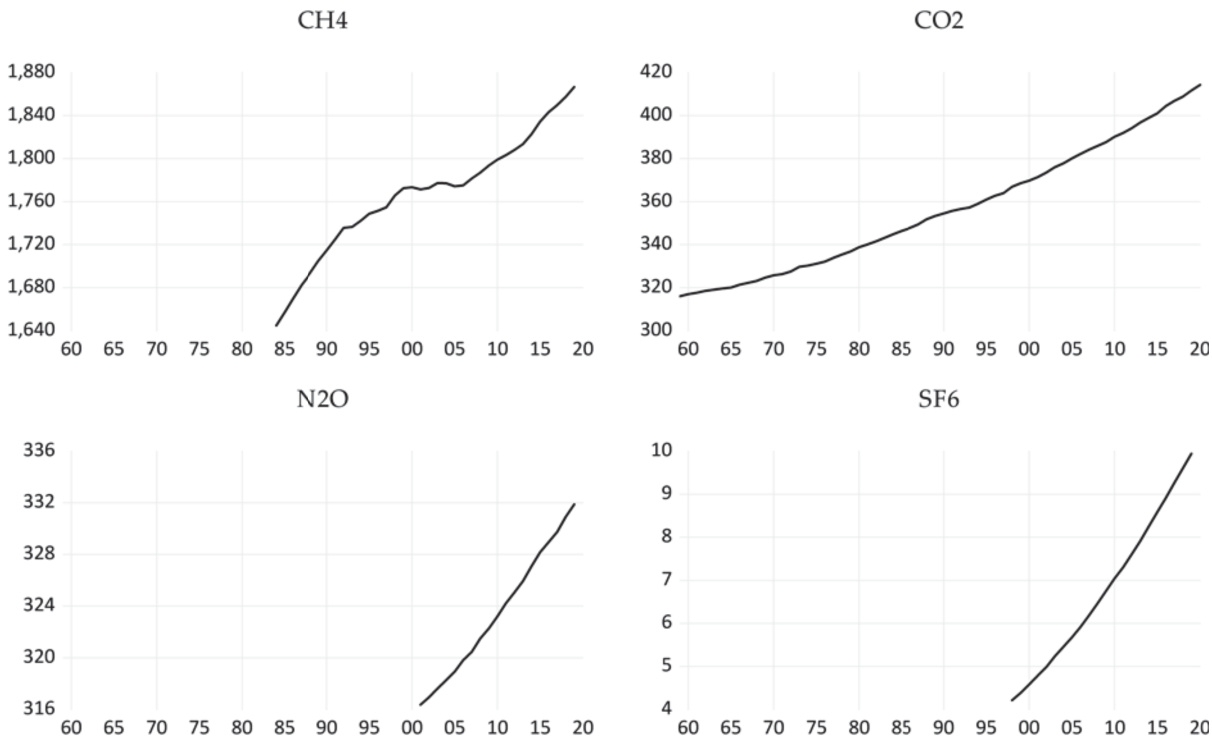
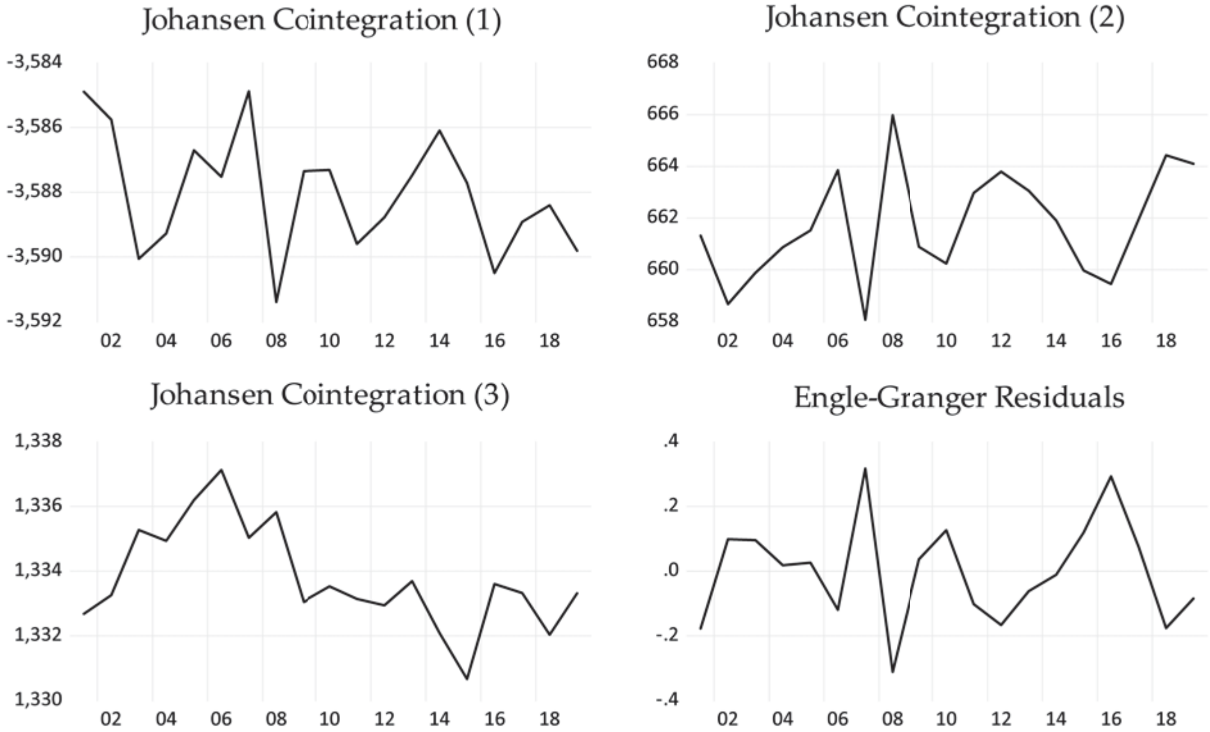

CH4
1,880
1,840
1,800
1,760
1,720
1,680
1,640
60 65 70 75 80 85 90 95 00 05 10 15 20
CO2
420
400
380
360
340
320
300
60 65 70 75 80 85 90 95 00 05 10 15 20
N2O
336
332
328
324
320
316
60 65 70 75 80 85 90 95 00 05 10 15 20
SF6
10
9
8
7
6
5
4
60 65 70 75 80 85 90 95 00 05 10 15 20

Figure (22)

Multivariate Cointegration Relationships



Data

	CO2	CO2_BP	Temperature	North Hem. Temp	South Hem. Temp	Fossil	Population
1959	315.98	NA	0.16	0.12	-0.06	NA	NA
1960	316.91	NA	0.04	0.08	-0.13	NA	3.03E+09
1961	317.64	NA	0.13	0.09	0.03	NA	3.09E+09
1962	318.45	NA	0.12	0.13	-0.07	NA	3.15E+09
1963	318.99	NA	0.06	0.14	-0.03	NA	3.21E+09
1964	319.62	NA	0.01	-0.19	-0.21	NA	3.27E+09
1965	320.04	11207.7	-0.07	-0.14	-0.07	753.69	3.34E+09
1966	321.37	11725.3	-0.05	-0.02	-0.09	815.86	3.41E+09
1967	322.18	12084.7	-0.09	0.03	-0.08	870.43	3.48E+09
1968	323.05	12743.1	-0.2	-0.07	-0.09	944.71	3.55E+09
1969	324.62	13530.9	-0.09	-0.03	0.14	1032.26	3.63E+09
1970	325.68	14312.9	0.14	-0.04	0.09	1119.06	3.7E+09
1971	326.32	14788.4	0.01	-0.15	-0.02	1191.82	3.78E+09
1972	327.46	15495.5	-0.24	-0.18	0.2	1257.22	3.85E+09
1973	329.68	16345.1	0.28	0.1	0.23	1319.19	3.93E+09
1974	330.19	16255.8	-0.19	-0.18	0.03	1345.44	4E+09
1975	331.12	16281.7	0.11	-0.06	0.02	1346	4.08E+09
1976	332.03	17173.1	-0.02	-0.21	0.01	1425.29	4.15E+09
1977	333.84	17739.3	0.13	0.12	0.24	1472.14	4.23E+09
1978	335.41	18016.1	0.16	0.02	0.12	1530.68	4.3E+09
1979	336.84	18596.5	0.15	0.08	0.25	1618.6	4.38E+09
1980	338.76	18433.6	0.33	0.17	0.35	1626.97	4.46E+09
1981	340.12	18202.3	0.51	0.37	0.27	1639.11	4.54E+09
1982	341.48	18022.4	0.14	0.05	0.23	1643.39	4.62E+09
1983	343.15	18185.1	0.53	0.25	0.38	1668.35	4.7E+09
1984	344.85	18852.4	0.3	0.04	0.27	1794.46	4.79E+09
1985	346.35	19249.9	0.22	0	0.23	1834.09	4.87E+09
1986	347.61	19579.1	0.31	0.13	0.23	1854.61	4.96E+09
1987	349.31	20186.5	0.32	0.25	0.41	1946.66	5.06E+09
1988	351.69	20863	0.56	0.37	0.4	2034.4	5.15E+09
1989	353.2	21242.6	0.17	0.27	0.27	2115.75	5.24E+09
1990	354.45	21331.5	0.36	0.53	0.37	2177.31	5.33E+09
1991	355.7	21338.6	0.43	0.41	0.4	2225.6	5.42E+09
1992	356.54	21433.7	0.46	0.14	0.31	2236.72	5.5E+09
1993	357.21	21488.9	0.36	0.19	0.28	2256.49	5.59E+09
1994	358.96	21709.9	0.27	0.37	0.26	2272.25	5.67E+09
1995	360.97	21982.9	0.56	0.58	0.32	2346.34	5.75E+09
1996	362.74	22598.7	0.25	0.26	0.39	2457.33	5.83E+09
1997	363.88	22749.9	0.34	0.52	0.41	2447.99	5.91E+09
1998	366.84	22819.7	0.6	0.7	0.51	2488.54	5.99E+09
1999	368.54	23127.8	0.51	0.48	0.28	2555.62	6.07E+09
2000	369.71	23676.4	0.34	0.5	0.29	2653.13	6.15E+09

2001	371.32	24010.3	0.47	0.64	0.43	2689.17	6.22E+09
2002	373.45	24544.5	0.71	0.71	0.54	2766.6	6.3E+09
2003	375.98	25767.5	0.72	0.75	0.49	2849.18	6.38E+09
2004	377.7	27077.5	0.61	0.66	0.42	2961.31	6.46E+09
2005	379.98	28186.5	0.65	0.86	0.5	3045.18	6.54E+09
2006	382.09	29074	0.51	0.82	0.46	3125.17	6.62E+09
2007	384.03	30095.9	0.92	0.85	0.48	3249.85	6.71E+09
2008	385.83	30378.4	0.27	0.68	0.4	3320.41	6.79E+09
2009	387.64	29745.2	0.6	0.73	0.59	3253.58	6.87E+09
2010	390.1	31085.5	0.73	0.9	0.54	3485	6.96E+09
2011	391.85	31973.4	0.47	0.75	0.46	3570.34	7.04E+09
2012	394.06	32273.5	0.44	0.82	0.48	3658.03	7.13E+09
2013	396.74	32795.6	0.62	0.8	0.56	3717.46	7.21E+09
2014	398.87	32804.7	0.7	0.92	0.57	3741.29	7.3E+09
2015	401.01	32787.2	0.83	1.18	0.62	3819.05	7.38E+09
2016	404.41	32936.1	1.12	1.31	0.72	3901.41	7.47E+09
2017	406.76	33279.5	0.98	1.18	0.67	4003.93	7.55E+09
2018	408.72	34007.9	0.76	1.04	0.66	4201.94	7.63E+09
2019	411.66	34169	0.94	1.22	0.75	4280.09	NA
2020	414.24	NA	1.15	1.36	0.68	NA	NA

[i] Furthermore, the time series can be fractionally integrated, whereby d in $(1 - L)^d y_t$ is less than one. If $d < 0.5$, y_t is said to be long-memory stationary and if it is > 0.5, it is said to be long-memory non-stationary. We could still make inference in regressions if the two time series have unit roots, but cointegrated, i.e., have a common trend. Further, similarly if they are fractionally integrated and fractionally cointegrated. Such findings may indicate that temperature and CO2 share a long –run common trend. We do not pursue this test because we will show that CO2 is I (1).

[ii] BLUE is Best, Linear, Unbiased Estimator.

[iii] The bandwidth parameter is l for the kernel-based estimators of f_0, which is the Newey-West (1994). They use AR1. So we choose the lag length p to minimize these criteria AIC $-2\left(\frac{l}{T}\right) + 2k/T$); the SIC $-2\left(\frac{l}{T}\right) + kln(T)/T$; HQ $-2\left(\frac{l}{T}\right) + 2kln(\ln(T))/T$. The modifications add τ to every k and $\tau = \alpha^2 \sum_t y_{t-1}^2 / \sigma_\varepsilon^2$.

[iv] The Ng – Perron (2001) test, which is a modified Phillips – Perron, two test statistics Z_α and Z_t, Bhargava (1986). These last two tests reject the unit root in the temperature data with two lags in the model, spectral GLS – de-trended AR based on AIC with maximum lag of 10. Different methods of estimating the spectral do not alter the results.

[v] We estimated and SVAR. Estimating an SVAR does not alter the results, therefore, we do not report the result. The results are available on request. The observed residuals e_t have a covariance matrix

$\sum(ee^{'})$. The structural VAR model is $Ae_t = Bu_t$, where $u_t$is a matrix of unobserved shocks, which we want to identify. This matrix has an identity covariance matrix $\sum(uu^{'}) = I$. Different methods can be used to identify shocks, but the orthogonality of the shocks implies that the identifying restrictions on A and B are of the form $A\sum A^{'} = BB^{'}$. Since the matrices on both sides of the equality sign are symmetrical, we have $k(k+1)/2$ restrictions on the $2k^2$ unknown elements in A and B. To identify A and B, additional $2k^2 - (k+1)/2$ identifying restrictions are needed. We use short-run restrictions on B. These restrictions imply that CO2 growth is unaffected by temperature, and it is a function of its own past only. Temperature, however, depends on its own lags and lagged CO2 growth rate.

[vi] Tests for the lag structure are based on a Wald – Chi Squared test. We run a 6-year lag VAR, but we find 3 to 2 lags to be significant. The joint P values indicate non-rejections. The lag-length tests include sequential modified LR statistics at 5% level; final prediction error, AIC, SIC, and HQ information Criteria. We choose 2 lags because temperature is volatile and affects the variance decompositions. The F stats for the equations in all VARs reject the hypothesis that the coefficients are insignificant. The residuals are tested for serial correlation using LM test, which cannot reject the null hypothesis of "no serial correlation" at lag 1 to 4.

[vii] Cooley and LeRoy (1985) famous article criticizing the *atheoretical* VAR method seems like a logical criticism.

[ix] Assume the following model

$$y_t = \alpha^{'} x_t + u_{ot}, \qquad (1)$$

and,

$$x_t = x_{t-1} + u_{xt}. \qquad (2)$$

Let $u_t = (u_{0t}, u_{xt}^{'})^{'}$ and assume that $u_t$is stationary and Gaussian with zero mean and spectral density $f_{uu}(\lambda)$ with$f_{uu}(0) > 0$. The cointegration relation (1) can be efficiently estimated by an empirical leads and lags regression of the following type:

$$y_t = \alpha^{'} x_t + \sum_{j=-k}^{k} \rho^{'} \Delta x_{t+j} + u_{kt} \qquad (3)$$

The lag and lead truncation parameter k satisfies$k \rightarrow \infty$, and$k^3/n \rightarrow 0$ as the sample size. Phillips and Loretan (1991) note that, in practice, it is useful to augment regression formulation in (3) with lagged equilibrium relation regressors that help to whiten the error term u_{0t} in (1) with respect to its own history. This leads to an empirical leads and lags and equilibrium lags regression equation:

$$y_t = \alpha^{'} x_t + \sum_{j=-k}^{k} \rho_j^{'} \Delta x_{t+j} + \sum_{s=1}^{r} \varphi_j (y_{t-s} - \alpha^{'} x_{t-s}) + u_{krt} \qquad (4)$$